26 June 2017 | Draft

# Interplay of Sustainable Development Goals through Rubik Cube Variations

## Engaging otherwise with what people find meaningful

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## Introduction

In a period of global crises a frame of reference is offered by the 17 Sustainable Development Goals articulated by the United Nations, superceding the earlier formulation of the 8 Millennium Development Goals. The new set of goals, as with the earlier set, is variously held to be beyond useful criticism -- being effectively "set in stone", as with the Babylonian Code of Hammurabi. Any reference as to the reasons by which the previous set has faded into history is carefully avoided -- irrespective of whether this might guide reflection on the possible early fate of the new set.

The concern here follows from the conclusion of an earlier argument (*Refining the Value of Sustainable Development Goals: in quest of the systemic coherence of global attractors*, 2017). The conclusion there focused on the possibility of organizing the 17 SDGs in terms of mathematical "magic squares", with all these imply in terms of pattern resolution (*Unexplored cognitive implications of 17 Sustainable Development Goals -- through "magic squares"?* 2017).

The argument there is that collective global choice of a disparate set 17 value-goals must necessarily be taken seriously. As noted by *Wikipedia*, the number 17 has wide significance in pure mathematics, as well as in applied sciences, law, music, religion, sports, and other cultural phenomena. It is potentially valuable to note the appreciation of 17 in mathematics, especially in the light of the insights offered by (George Lakoff and Rafael Nuñez, *Where Mathematics Comes From: how the embodied mind brings mathematics into being*, 2001). Other than as a fantasy of ball-game enthusiasts (as noted there), why does 17 "work"? Missing is any insight into how the collective decision-making process elicited 17, or why the number was perceived by some to constitute a coherent set.

The earlier argument made particular reference to Rubik's Cube, its more complex variants, and their implementation in virtual reality applications as a means of bypassing technical and other constraints of physical construction and distribution. It is appropriate to recognize the considerable development of the cube user community worldwide. The focus of the World Cube Association (WCA) is on speedcubing and the regulation of speedsolving competitions for Rubik's Cube and similar puzzles.

The value of using such cubes for personal cognitive development and the enhancement of thinking capacity has been specifically articulated and studied (*What are the benefits to solving Rubik's Cubes? Quora*, 2015). Whilst appreciating that puzzle-solving focus, **the concern here is with how use of such devices might be adapted to enhance pattern recognition of relevance to enabling sustainable governance**.

Beyond the technicalities of such adaptation, of particular interest is the obvious appeal of such a device over decades, readily defined as "magical" -- whatever that may mean (William Lee Adams, *Puzzling Success*, *Time*, 28 January 2009; Ariel Sabar, *Behind the Unceasing Allure of the Rubik's Cube*, *Smithsonian Magazine*, July 2014). It is in relation to that magical appeal that the case is made here for the relevance of the extensive insights into "magic squares".

Potentially relevant to sustainable development is the familiarity of users with the skills and insights into the variety of "algorithms" by

which a cube may be solved through a sequence of operations: left-right, up-down, front-back (see example: *Algorithms to Solve Rubik's Cube*). Given the metaphorical use of those terms in the politics of strategic development, this offers a new way of thinking about analogous operations in psychosocial systems for the resolution of the challenges of global governance.

**What might be the algorithms of sustainable development?** Through the metaphors, could these be inspired by *Wikipedia*'s summary of *Optimal solutions for Rubik's Cube*? However it is not just the challenge of comprehending those algorithms, as helpfully articulated by Tomas Zaremba (*The Easiest Way to Memorize the Algorithms of Rubik's Cube*, *Instructables*, 2017). Rather it is the cognitive challenge of understanding the implications in psychosocial systems which is the concern here, following from the argument of Douglas Hofstadter and Emmanuel Sander (*Surfaces and Essences: analogy as the fuel and fire of thinking*, 2013).

## Recognition of Rubik's Cube as a relevant strategic development metaphor

The metaphor offered by Rubik's Cube was used by Heiner Benking to develop a 7x7 "*Rubik's Cube of Ecology*" in 1990 (as part of the Global Change touring exhibition) as a means of explaining the nature of integration and of developing a framework for co-ordination and harmonisation across various fields, areas, regions, disciplines and domains (*Visual Access Strategies for Multi-Dimensional Objects and Issues*, WFSF/FAW, 1993; *The Hyperlink ECO-CUBE: a central piece of a knowledge panorama for clearinghouse organisation and (environmental) education*, 1994). It was designed to offer a new world view, using a hyperlinked Eco-Cube, for better understanding and communication about multi-disciplines like ecology.

**Views of the Eco-Cube**
(as developed by Heiner Benking)



As discussed separately (*Pointers to possibilities: Ekistics, Dymaxion World and Eco-Cube*, 2010). the Eco-Cube is reminiscent of the Ekistics Grid developed by Constantinos Doxiadis as a framework for organizing information relevant to ekistics as the science of human settlements. This has been used to further develop ekistic concepts, and also in the application to practical problems. Such a grid display any component within two dimensions at a point of intersection of abscissa and ordinate The abscissa of ekistic units remained constant in all uses of the ekistic grid, and the most usual ordinate consisted of the five ekistic elements: Nature, Anthropos (Man), Society, Shells (dwellings or buildings), and Networks, with a sixth line denoting their Synthesis.

Extensive use of Rubik's Cube as a metaphor has recently been made with regard to strategies relating to sustainable development -- but with little effort to construct such a cube as a guide to what it might enable:

- ***climate change***: In a description of the UN Climate Change Conference, Mike Hulme argues: *The world arrived at Copenhagen with a Rubik's cube climate-change puzzle containing just too many dimensions to be solved* (*Moving Beyond Climate Change*, *Environment: science and policy for sustainable development*, May-June 2010). See also: Gregory Briner, *Mitigation: Solving the Rubik's cube of climate change*, *OECD Observer*, November 2015; Steve Waters, *Contingency Plan and the Rubik's Cube of climate change*, *The Center for Sustainable Practice in the Arts*, 20 November 2009.

- ***water cooperation and security***: Anders Jägerskog et al. (*Transboundary Water Cooperation: a Rubik's Cube*, 2013) and V. Stucki, et al. (*Water and security in Central Asia: solving a Rubik's Cube*, 2013)

- ***health***: A presentation to a Global Health Practitioners Conference (theme: *Achieving Health for All by Multidimensional Approaches*) by Ester Indriani and Lee Po Cha (*Multi-dimensional Programming: the Rubik's Cube challenge to community health*, Portland, 2016) argues that so many models and frameworks have been tried, but concludes that these are "too flat" to capture the dynamics, relationships, and movement. Arguing for a Rubik-style cube, the question is how has it been solved/built, what was missed, and how does it looks to you: what are the facets of your cube?

- ***women and children***: The "SDGs Rubik's Cube" featured as a metaphor in a UN High-level Political Forum on the theme *Every Woman Every Child* (New York, July 2016) with respect to the *Global Strategy for Women's, Children's and Adolescents' Health* and related Sustainable Development Goals. The panelist, Margaret Batty, subsequently argued the relevance in some detail (*The Rubik's Cube of the UN Global Goals*, *The Huffington Post*, 29 August 2016).

- ***sustainable food security***: As proposed in a course promoted as *Learn how to solve the 'Rubik's cube' of systems thinking and how it's applied to improve the environmental sustainability of food production systems* (*Sustainable Food Security: the value of systems thinking*, Wageningen University)

- ***poverty alleviation***: As described in an interview, new models put together poverty, environment, and culture using the metaphor of a Rubik's cube to illustrate all the different options (Steven J. Lade, et al. (*Resilience offers escape from trapped thinking on poverty alleviation*, *Science Advances,* 3, 2017)

- ***communication systems***: At a conference on broadband communication in Europe, in the country from which Rubik's Cube originated, cube offered during the conference as a gift to every participant was seen as a great image applicable for broadband development in Europe. As noted in that respect: *Europe has all the items to be successful in Broadband development for the future, but the key is how to associate and foster in the right way all the different elements in order to avoid to get a unassembled patchwork. An inadequate management of potential options and solutions has the serious risk to make Europe loose the Broadband train for the future.* (*Broadband for all: the right Rubik's cube to achieve*, *Telefonica*, 11 March 2011)

- ***sustainable economy***: As noted by Adam Stones: *... the systems that guide our lives - whether that's a city, a society or sector - are like the coloured squares on a Rubik's cube. Try and fix one of them that is out of place alone and you will never solve the problem; in fact you will cause unintended disruption elsewhere. You have to look at the whole puzzle. Even the UN's Sustainable Development Goals (SDGs) can only become effective if we apply systems thinking* (*Is this the year the world wakes up to the Circular Economy?* *Metabolic Quarterly*)

- ***environmental sustainability***: For J. B. Ruhl: *Working on sustainable development will be difficult, tricky, and frustrating. Like the Rubik's cube, when it looks like one side of sustainable development is coming together, we may find the other sides are no further along or even more out of order. Unlike the Rubik's cube, however, there is no end point for sustainable development -- we don't "solve" it. At best, we develop policy approaches that keep the cube in play* (*Law for Sustainable Development: Work Continues on the Rubik's Cube*, *Tulsa Law Review*, 44, 2013, 1).

- ***land degradation***: *Multiple demands, multiple perspectives, one fixed land base. It's like holding a Rubik's Cube. A jumble of multiple demands and perspectives on land. Right at a time when foreign investment in land is booming. When food security is a recur ring concern. When land can provide around 40% of near-term climate change mitigation. We have to work on the whole Rubik's Cube. There's no point getting one face of the cube nicely aligned with the same colour if in the process you mess up all the other faces. And it's going to get more difficult in the decades ahead. We can no longer think only at the national scale. In a globalised and shrunken world, global land use must add up to less than one planet*. (*The Rubik's Cube of Land* cited in *Zero Net Land Degradation: a sustainable development goal to Rio+20*, UNCCD Secretariat Recommendations for Policymakers, May 2012)

- ***sustainable business***: James Crowley and Cristina Alzaga (*Connecting Business and Development: the "Rubik's Cube" of cross sector collaboration*, Crowley Institute, 2009)

- ***tax reform***: With the aid of a surprising animation, Neil Irwin argues: *What makes trying to solve a Rubik's Cube so exasperating is that every rotation you make to align the colors on one side messes up something on one of the other sides. Nothing moves in isolation; everything affects everything else, and rarely for the better*. (*Can Trump and Congress Solve the Rubik's Cube of Tax Reform?* *The New York Times*, 11 April 2017)

- ***inequality and power***: As summarized by its originators: *One of the unique characteristics of the power cube approach is the interactivity of its various dimensions, where configurations of power are shaped by the interplay of the forms, levels and spaces of power. As argued previously: '[W]ith this more complex approach, the three dimensions of power elaborated by Lukes may be seen as three forms of power along a single dimension or continuum. By thinking of the levels of power and the spaces of power also as dimensions, or continua, each of which interacts with the other, we can visually understand power as a sort of Rubik's cube'... which we have called the power cube.* (John Gaventa and Bruno Martorano, *Inequality, Power and Participation: revisiting the links*, *IDS Bulletin*, 47, 2016, 5)

- ***urban quality of life***: (Alexander G. Keul, et al, *Urban Quality of Life: a Rubik Cube of objective and subjective descriptors*, *Momentum Quarterly*, 6, 2017)

- ***human psychology***: A critique of the fragmentation of psychology as a discipline argues that: *Carving up psychology at its joints results in a Rubik's Cube of 27 subfields* (Axel Cleeremans, *The Grand Challenge for Psychology: Integrate and Fire!* *Frontiers in Psychology*, 2010). The metaphor has been used with respect to the social scinces in general (Herman van der Merwe, *Social sciences: the Rubik's Cube of humanity*, *Northwest University News*, 29 March 2017).

- ***terrorism***: Citing an earlier study using the image of Rubik's Cube, Diane Perlman notes: *If we focus on solving one facet of the problem, trying to get one side of the cube all red, for example, we actually set back the solution of the problem as a whole. Our intense approach to counterterrorism is like working on one facet of the Rubik's Cube* (*The Psychology of Terrorism: a public understanding*, 2002, p. 36). And, in a similar vein, Richard North Patterson (*North Korea is Trump's nuclear Rubik's Cube*, *The Boston Globe*, 21 March 2017)

- ***innovation***: *The Manifesto for Creativity and Innovation in Europe* is considered one of the key outcomes of the European Year of Creativity and Innovation 2009 (*Rubik's cube and EU Politics: the Manifesto for Creativity and Innovation in Europe*, *Europa,* 12 February 2010). Emblematic of that creativity, a cube was presented to the president of the European Commission by its inventor Ernö Rubik who had been named an Ambassador of the Year.

The potential of Rubik's Cube in relation to SDGs has been recognized by a UN-sponsored initiative in Tblisi (*UN unveils art installation*

*to take SDGs public*, *United Nations in Georgia*, 24 October 2016). This featured a large-scale Rubik's Cube with icons of the SDGs affixed to its surfaces. Through this project, launched on UN Day, the UN Office in Georgia aimed to raise public awareness of the Sustainable Development Goals. The press release argued that: *Like the Rubik's Cube, the Sustainable Development Goals are universal, indivisible and linked to each other*. The image below was circulated by the network of UN Information Centres.

**UN Art Installation in Tbilisi, Georgia**

Reproduced from *UNIC@Work*, October 2016 - Special UN Day Bulletin

The question is whether reference to Rubik's Cube can be taken beyond rhetorical metaphor, even evident in the most prestigious contexts (Ron Ashkenas, *Solving the Rubik's Cube of Organizational Structure*, *Harvard Business Review*, 15 March 2011). For example, an award-winning experiment has been undertaken to demonstrate the use of Rubik's Cube as an augmented reality (AR) game platform in support of the UN Millennium Development Goals, as described by the authors:

> To explore Rubik's cube as an AR game platform, we designed and implemented two games.... a puzzle game, which is organized in levels. The game was designed to advance the United Nations Millennium Development Goals... with the premise of aiding in the development of poor countries. In the game, virtual villages are augmented on the cube faces. These villages develop according to the resources they receive which are controlled by the arrangement of the cube. The goal is to reach equilibrium of resources across all villages. To achieve this, the player shu?es virtual assets between the villages (Oriel Bergig, et al., *Out of the Cube: augmented Rubik's Cube*, *International Journal of Computer Games Technology*, 2011)

As noted by Don Saari in a lecture at the International Institute for Applied Systems Analysis (IIASA):

> The danger of reductionism... is that it can turn out completely incorrect solutions -- without any indication that they are incorrect. He said, "The whole may be completely different than the sum of its parts". Take a Rubik's cube as an example: Saari said "If you try to solve it by first doing the red side, then the green, then the blue, you will end up with a mess. What happens on one side is influenced by what's happening on all the other sides". In the same way, the world's great systems of energy, water, climate all influence each other. (*Charting connections: the next challenge for systems analysis*, *Nexus*, 4 September 2014)

The difficulty with this assessment is that charting systemic connections does not render them comprehensible or meaningful in the sense required for empowering action in relation to sustainable development goals. The point is well made by the well-charted connections in relation to climate change -- and the indifference to which their recognition gives rise. Can this recognition be taken further through deeper understanding of the fascination with pattern recognition -- evident to a degree in the case of Rubik's Cube, but so questionable in the case of global promotion of sustainability goals.

The possibility of *Rethinking Rubik's Cube: a mnemonic device for ways of knowing and engagement?* has previously been discussed within the context of *Existential Embodiment of Externalities: radical cognitive engagement with environmental categories and disciplines* (2009).

## Elusive requisite "magic" for sustainability?

No "magic", "no sustainability"? How is any "magic" to be recognized -- however it may be associated with sustainability? Is there every possibility that it would be unrecognizable to many -- even when faced with it?

**Economic magic?** It is noteworthy that some use of a (2x2) "magic square" has been made in relation to sustainable development by Andreas Brunold (*Global Learning and Education for Sustainable Development*, *Higher Education in Europe*, 30, 2005, 3-4; *Civic Education for Sustainable Development and its Consequences for German Civic Education, Didactics and Curricula of Higher Education*, *Discourse and Communication for Sustainable Education*, 6, 2015). As a pattern, such usage follows a particular tradition in Germany (Sebastian Dullien, *A New "Magic Square" for Inclusive and Sustainable Economic Growth: a policy framework for Germany to move beyond GDP*, Friedrich-Ebert-Stiftung, 2017). Peter Bartelmus notes that:

> Macroeconomic policies have traditionally focused on the achievement of overall objectives such as the "magic square" of economic growth, full employment, balanced foreign trade, and price stability (*Environment, Growth and Development: the concepts and strategies of sustainability*, 2002, p. 85)

**Magic of coherence?** Whilst any use of the term "magic" can be readily deprecated, it is important to recognize the value it has in

indicating subtle attraction -- if not fascination. How is the sense of coherence offered by the quest for it to be associated with "magic"? In an obvious sense it is readily recognized in the attraction offered to so many by completion of a puzzle, as in the case of Rubik's Cube, sudoku or crossword puzzles -- all of which are a focus of worldwide fascination.

The improbable balance elicited by their resolution is even evident in the recognition by nuclear physicists of a near magic number of protons and neutrons which would temporarily reverse the trend of decreasing stability in elements heavier than uranium, resulting in an "island of stability" -- one appropriate metaphor for global sustainability. Is it surprising that puzzle-solving and game-playing by the wider population, and the young, are now anticipating the quest for global strategic order by elites?

**SDG magic?** How is the fascinating attraction of Rubik's 3x3x3 cube -- acknowledged as the world's most popular puzzle game and toy -- to be compared with engagement with the SDGs? Why has the cognitive challenge so framed been extended to Rubik's Revenge (a 4x4x4 variant), the Professor's Cube (5x5x5), the V-Cube 6 (6x6x6), the V-Cube 7 (7x7x7) and the V-Cube 8 (8x8x8)?

Given the continuing fascination with Dürer's magic square in the engraving *Melencolia I* (1514), it is appropriate to recognize the potential significance of its 3 dimensional variants (*Magic cube with Dürer's square*, 14 October 2009; *Dürer's square*, *Complex Projective 4-space*, 8 September 2012). Of related relevance (as discussed below), the fascination also extends to the polyhedron depicted in that engraving, especially the manner in which a portion of it is hidden -- rendering the structure and significance of the whole the subject of extensive commentary.

Is it possible that the cognitive patterns of organization and engagement relevant to future governance are being anticipated in modes of puzzle-solving, game-playing and music which are currently dissociated from appreciation of that relevance? Is there a curious "cognitive disconnect" inhibiting "joined-up thinking"?

The interest of Freemasonry in magic squares has been widely noted in relation to its purported role in constitutional development (notably in the USA), as well as in its particular interest in the Dürer magic square, as discussed separately (*Transformational pathways between 2D and 3D in the light of graph theory and "magic"*, 2016; *Salvation Enabled by Systemic Comprehension Via aesthetics of magic squares?* 2015).

**Magical construction**: The quest for "magic" frames a desirable interplay of value-goals as potentially to be associated with **"islands of cognitive stability" through which sustainable global development might be comprehended as both credible and attractive.** In the light of the above argument, there is a case for confronting the various cognitive constraints and possibilities highlighted by George Miller's "*magical number seven, plus or minus two*" (1956) with the "*where mathematics comes from*" (2001) of George Lakoff and Rafael Nuñez. Do these variously frame the quest for "magic" in terms of 2x2 (as in macroeconomics), 3x3 (as in the Millennium Development Goals), or 4x4 (as with the Sustainable Development Goals)?

The first is suboptimally challenged by the quintessential (Betty Cornfeld and Owen Edwards, *Quintessence: the quality of having it*, 1983; Peter Senge, *The Fifth Discipline: the art and practice of the learning organization*, 1990). Are the 8 Millennium Development Goals to be considered as similarly suboptimal -- constrained by cognitive failure to embody the ninefold?

Is the challenge of configuring the 17 Sustainable Development Goals an exercise in doing just that -- especially given the pattern of 9-fold and 8-fold axes implied by the tetrahedral geodesic sphere as an exemplification of the global brain? Especially noteworthy is the role attributed to the ninefold by Freemasonry, as discussed separately with respect to the nonagonal symbol of its penultimate degree (*Speculation on Potential Symbolic Relevance of the Concordian Mandala*, 2016).

## Mapping 8 Millennium Development Goals onto the 3x3x3 surfaces of Rubik's Cube

Any mapping exercise involves a particular distortion resulting from the choice of projection (see *List of Map Projections*, *Wikipedia*). The question is how to take advantage of the obvious credibility and attraction of the Rubik Cube to explore ways of handling the 8 Millennium Development Goals -- whether or not those are understood to have been superceded.

**Requisite nothingness**: The 3x3 subdivision of each face of the cube suggests that one approach is to treat the central position differently, as might be variously justified -- if only in the technology of the cube itself. Philosophically it may be usefully explored as a necessary central " hole", as remarkably discussed by Roberto Casati and Achille C. Varzi (*Holes and Other Superficialities*, 1994) -- with respect to the borderlines of metaphysics, everyday geometry, and the theory of perception (as they summarize in the entry on holes in the *Stanford Encyclopedia of Philsophy*). Such a configuration could also be justified through recognition of the particular cognitive role of the "missing", as articulated by Terrence Deacon (*What's Missing from Theories of Information?* 2010). Given the fundamental preoccupation of the MDGs with the "nothingness" that is tragically central to the lives of so many, its reflection in any configuration of MDGs is especially appropriate.

The point has been made otherwise in the widely-appreciated insight of Taoism, as applicable to 8 "spokes" (or 17), as to thirty.

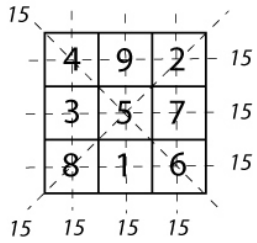
| *Tao Te Ching* by Lao Tzu<br>Chapter 11 |
|---|
| *Thirty spokes share the hub of a wheel;*<br>*yet it is its center that makes it useful.*<br>*You can mould clay into a vessel;*<br>*yet, it is its emptiness that makes it useful.*<br>*Cut doors and windows from the walls of a house;*<br>*but the ultimate use of the house*<br>*will depend on that part where nothing exists.* |

| |
|---|
| *Therefore, something is shaped into what is;*<br>*but its usefulness comes from what is not.* |
| Translated by Kari Hohne, 2009; other translations |

**Ordering the array**: The approach would indeed allow the face of the 3x3 cube to encompass 8 "goals". An obvious question is then how best to order them to suggest and enhance their meaningful interplay.

The pattern of 8 symbols, with an empty centre, is highly reminiscent of the evocative *BaGua* arrangement of 8 symbols -- whether denoted by trigrams or idiograms. Framed as a *BaGua* mirror, it is traditionally associated with the "magic" of feng shui, predating the association of those symbols with the *I Ching*. It has a particular advantage in relation to value-goals in that it has strong qualitative connotations with personal implications. The implications of that pattern are dicussed further below.

As an exercise, the pattern of 8 MDGs could be ordered in an array corresponding to the numbering of the magic square on ther left. Clearly this fails because the numbering does not extend to 9. Some such ordering would be possible if the MDGs were renumbered and particular significance was attributed to the central role of 5 in the pattern (as suggested by the verse above) -- and as is done with the 17th SDG.

| **Relevance of a "magic square" configuration of Millennium Development Goals?** | | | |
|---|---|---|---|
| **3x3 Magic square**<br>(originally described by Lo Shu) | **Towards an arrangement of Millennium Development Goals around an empty centre** | | |
|  |  |  |  |
| | no change | change to 9 | no change |
| |  | *5* |  |
| | no change | add 5 | change to 7 |
| |  |  |  |
| | change to 8 | no change | change to 6 |

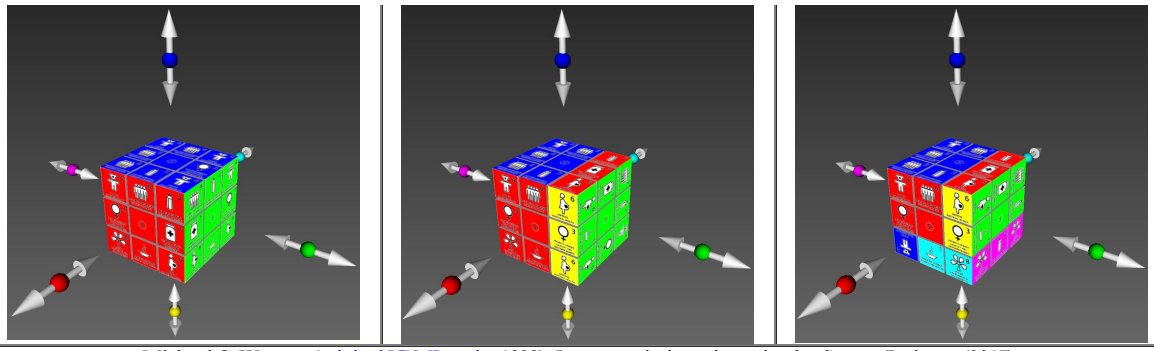
The question would then be how to make meaningful use of all six faces of the cube -- with 54 (6x9) surfaces. Are there one or more arrangements of the 8 goals on the faces of a cube which can be construed as charactwristic of greater systemic integrity as a consequence of the characteristics of the magic square ad the manner in which its different arrangements meet at the edges, the corners, and/or preserve the magic constant of 15?

| **Lo-Shu Magic Square and its rotations and reflections** | | | |
|---|---|---|---|
| Lo-Shu Magic Square | 90 degree rotation | 180 degree rotation | 270 degree rotation |
| 4 9 2<br>3 5 7<br>8 1 6 | 2 7 6<br>9 5 1<br>4 3 8 | 6 1 8<br>7 5 3<br>2 9 4 | 8 3 4<br>1 5 9<br>6 7 2 |
| Reflections of the above through the centre column | | | |
| 2 9 4<br>7 5 3<br>6 1 8 | 6 7 2<br>1 5 9<br>8 3 4 | 8 1 6<br>3 5 7<br>4 9 2 | 4 3 8<br>9 5 1<br>2 7 6 |
| Reproduced from *Magic Squares of Order 3* where it is noted that reflections through the center row give no new arrangements, as can be readily verified. Likewise, no reflection through a diagonal will yield a new square. Further considerations are apparent in the discussion of *Magic cube classes*. | | | |

Part of the challenge is that a cube has only 6 sides, whereas 8 possible configurations are indicated above, whose contiguity in 3D might be understood as suggestive of patterns of a higher order of integrity -- if only in terms of memorability. Two of the configurations may well be redundant for that purpose -- or as offering alternative understandings of systemic integrity. This is then an exercise in puzzle solving to identify the most integrative solution(s). Another approach might be to consider a mapping of all 8 onto the surface of an octahedron -- as the dual of a cube -- again in the quest for the most memorable configuration.

In anticipation of enabling the puzzle solving challenge, an interactive virtual reality implementation of attributing MDGs to the surfaces of a Rubik's Cube has been adapted experimentally and illustrated below. The original VRML implementation was developed by Michael G. Wagner (*Application of quaternion algebra*, 1998; original VRML code). The adaptation to allow attribution of images was kindly developed by Sergey Bederov of Cortona3D (who is not responsible for the current attribution of MDGs to the cube).

| **Screen shots of Millennium Development Goals applied to interactive Rubik's Cube in virtual reality**<br>(***provisional*** attribution for indicative purposes only) | | |
|---|---|---|
| "resolved condition" ? | one twist (right face) | two twists (bottom face) |
| | | |



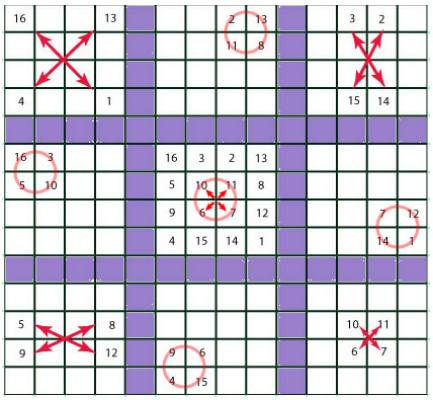
Michael G. Wagner (original VRML code, 1998); Image rendering adaptation by Sergey Bederov (2017)
(Version with MDG images ***provisionally*** added: VRML)
Given the evolution of rendering of VRML through web browsers, in its current form the interactive features operate without difficulty using the Cortona3D plugin on Internet Explorer, whereas other browsers may well be constrained with regard to most features. [The application can also be downloaded to run on local domains if intranet security is modified as described here]

The attributon of images is readily made in the source code use a text editor and further experiments will be undertaken towards demonstrating the sequence of rotations towards the "solution" of a scrambled cube.

## Magic square of sustainability: from 3x3 to 4x4?

The argument with respect to the 8 Millennium Development Goals can be understood within the framework of a 3x3 magic square -- a central square being omitted from consideration (as argued above). The 17 Sustainable Development Goals suggest the possibility of using a 4x4 magic square of which Rubik's Revenge is the clearest example. The argument of the earlier paper focused on the manner in which the 17 SDGs could be understood in relation to the properties of a a particular magic square with magic constant of 34, namely 2x17 as indicated below.

**4 x 4 magic square with magic constant 34**
(with indication of portions of the square at the centre which total to 17)



The number 34 also invites reflection in this context as the magic constant of a 4-by-4 normal magic square (namely with columns, rows and diagonals each adding to 34; as in the central square on the left).

Encompassing all the numbers less than 17, the pattern includes the number of value-goals distinguished as 8 Millennium Development Goals, as well as the 15 global strategic challenges identified by the Millennium Project.

That magic square is a notable feature of a much commented engraving by Albrecht Dürer (*Melencolia I*, 1514).

According to *Wolfram Mathworld*, Dürer's magic square has the additional property that the sums in any of the four quadrants, as well as the sum of the middle four numbers, are all 34. It is thus a gnomon magic square. As indicated on the left, any pair of numbers symmetrically placed about the center of the square sums to 17, a property making the square even more magical.

The number 17 figures variously in several magic squares held to be significant in other cultures, for example the Chautisa Yantra (India). That of Dürer is held to be very similar to Yang Hui's square, created in China some 250 years before Dürer's time.

On the assumption that the pattern of 17 value-goals was not fortuitous but was enabled by intelligence, whether conscious or unconscious, the first 16 can be distributed according to the above pattern -- if only as a preliminary mnemonic exercise. Potentially this would suggest an insightful pattern of particular relationships between the value-goals, as highlighted by those above totalling to 17. Appropriately perhaps, the excluded 17th value-goal is *Partnerships for the Goals*. It is important to emphasize that the examples (cited above) of use of the 4x4 presentation (especially as 4x4x4) do not endeavour to distribute the value-goals in any ways suggestive of the systemic connectivity with which any "magic" might be associated.

**Highlighting potentially supportive relationships between 16 value-goals**
through their distribution (as numbered) according to the 4x4 magic square pattern above

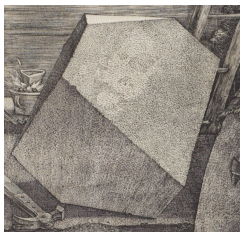
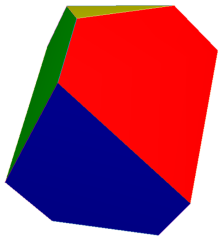
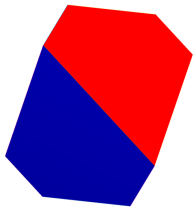

If use is to be made of the 4x4x4 Rubik's Revenge, the issue remains as how best to employ the other 5 faces of the 4x4x4 cube. The results of a very preliminary experiment are presented below.

In response to the question asked in the jargon of the cubing community: *Can anyone provide me with the algos of 4*4 Rubik's cube?* (*Quora*, 17 October 2015), a remarkably comprehensive checklist was indicated (*4x4x4 Parity Algorithms*, *Wiki Speedsolving*). This has the technical clarification: that parity algorithms occurring in a situation when only two or four edge pieces need to be cycled in order to complete solving the 4x4x4 or at least successfully bring the 4x4x4 into a pseudo 3x3x3 state. In that light, as asked above, **what might be the algorithms of sustainable development** represented in a 4x4x4 pattern, and what might be the appropriate "clarifications"?

## Beyond the superficial: engaging other sides as the hidden challenge of sustainability

**Comprehending the hidden**: A polyhedron like a cube offers the particular advantage that much of the structure is hidden from any given perspective -- as with any effort to perceive the planetary globe as a whole. This characteristic echos the case made with regard to an empty centre -- possibly understood as a hole in cognitive terms. There is necessarily always something missing from global comprehension. The point is usefully made with respect to the UN art installation depicted above. The other side(s) cannot be seen in the absence of some other operation -- waking around, etc.

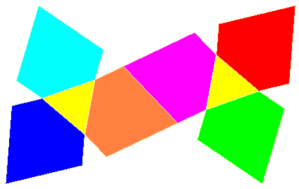
Given the commentary it has long evoked, the point is also clarified by the polyhedron in the engraving by Albrecht Durer's *Melencolia I* (as mentioned above and depicted below). The question has long been what was the actual form of that polyhedron, and the implications to be drawn from that. One strong assumption is that it is a complex of 6 pentagons associated with 2 triangles, as shown in the two images on the right.

| Clarifying the challenge of the "hidden" of relevance to a global perspective on SDGs | | | |
|---|---|---|---|
| UN art installation (detail of image above) | Dürer's polyhedron (detail of engraving) | Dürer's polyhedron (corresponding construction) | Dürer's polyhedron (rotated) |
|  |  |  |  |
| | | | |

**Illusion of 2D "comprehension" of coherence**: It is readily assumed that a "flat" array in 2D -- a map -- addresses the challenge of comprehension of global coherence. The point is partially made by unfolding the Durer polyhedron as in the image on the left (below). Such a presentation offers an illusion of clarity and comprehension regarding the nature of the structure. The nature of the illusion in the quest for strategic integration is evident to a degree from the dynamics of folding and unfolding the flattened array in the animation on the right (below). How that coherence is to be comprehended and enabled in practice is the case for any array of sustainable development goals.

Given any focus on a particular set of value-goals, as might correspond to the mandate of a particular UN Specialized Agency, what is partially or completely "hidden" from that focus and what implication does it have for a coherent understanding of the whole? More specifically, and of greater significance, is whether and **how "sustainability" is dependent on the coherent relationship between facets variously hidden from each other**. A polyhedron, as with Rubik-style cubes, is helpful in rendering this challenge more explicit.

| Clarification of assumed Dürer construction | |
|---|---|
| Representation of unfolded polyhedron | Animation of polyhedron (un)folding |
| | |



Images prepared using *Stella Polyhedron Navigator*

**Lure of the mysterious**: In emphasizing the attractive power of "magic" for many, the associated role of the "hidden" merits particular recognition. The challenge to enhancing the attraction of an array of sustainable development goals is how to indicate the significance of what is hidden and potentially emergent -- the hidden magic of that condition.

Why is sustainability less attractive to the popular imagination than the mysterious fictional focus given to the yearning for the "lost"? This is most obvious in the tales of the quest for lost treasure, lost knowledge, secret codes, or the Holy Grail -- exemplified by the cult following elicited by the novels of Dan Brown (*The Da Vinci Code*, 2004; *The Lost Symbol*, 2009).

**Millennium Development Goals**: One approach might then be to have 8 surfaces with icons on one side of a cube matched by 8 with icons on the opposite side. This could serve to encompass conceptually the highly probable problematic opposition to the original 8 goals -- themselves perceived as opposed by those of "problematic deposition". Each opposing side is then effectively a "shadow" of the other -- whether in a psychological sense or in that of ball-games when players on one side are expected to "shadow" or mark the players corresponding to them on the opposing side.

Whilst this approach takes account of two sets of 8 MDGs (on a single dimension), the issue is then what significance might be usefully attributed to the other two sets of 8+8 sides -- retaining the emptiness of the central square in each case. A valuable clue is offered by the colouring of Rubik's Cube. Six colours are necessarily required; one for each surface of the cube as a whole. Of interest is the use of the primary colours Red, Green and Blue -- potentially to be matched by Yellow, Mauve and Cyan.

A further clue is provided by the widely popular science fiction movie regarding strategic decision-making, namely *The Matrix* (1999), as discussed separately (*Psychosocial Transformation by "Pill Pushing"? Model-making, strategic advocacy and the myth of the "red pill"*, 2017). This notably features a choice framed in binary terms between the Red Pill and the Blue Pill, namely a choice between enabling knowledge, freedom and the sometimes painful recognition of reality -- in contrast with a blue pill, typically deprecated as reinforcing more-of-the-same, namely falsehood, security and blissful ignorance of illusion (Jenny C. Yip, *Red Pill or Blue Pill? What You Don't Know May Hurt You! Psychology Today*, 6 September 2012). The Red Pill can also be understood as associated with radical strategies, however these are appreciated or deprecated.

Arguably the binary nature of the choice obscures the possibility of a hypothetical Third Way -- or even a Middle Way. This can be usefully associated with the colour Green. The three primary colours, Red, Green and Blue can then be understood as opposed or shadowed by problematic perspectives -- from which they themselves would be appreciated as problematic (as noted above). The colours Yellow, Mauve and Cyan can be used to hold these understandings. Each side thus frames itself as "positive" and its opposite as "negative" -- as with the sides in any ball-game and the views vigorously held in that regard.

The 8 Millennium Development Goals then lend themselves to:

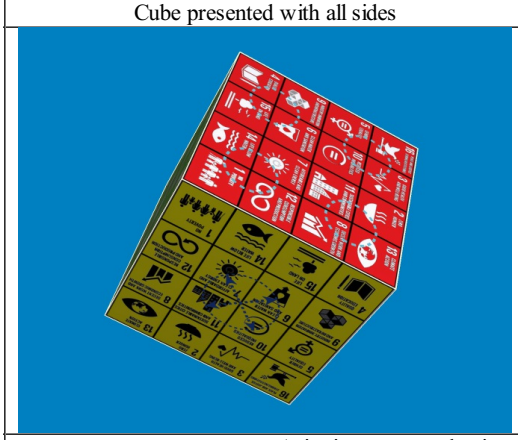
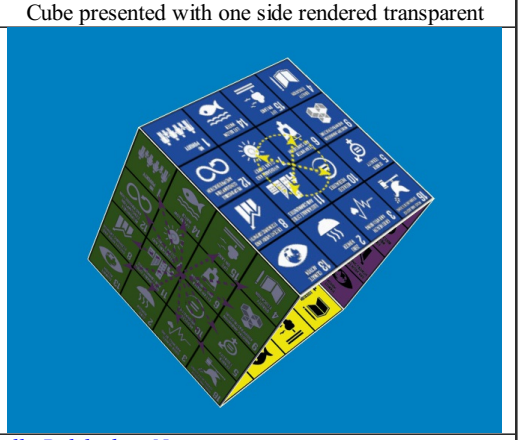
| | |
|---|---|
| a Red interpretation, namely of a more revolutionary (radical) strategic nature | a Cyan interpretation, opposing any radical approach and undermining its successful achievement |
| a Blue interpretation, namely of a conventional (adaptive) strategic nature | a Yellow intepretation, opposing any adaptive approach and undermining its successful achievement |
| a Green interpretation, namely any alternative to a binary configuration of strategic choices | a Mauve interpretation, opposing any alternative approach and undermining its successful achievement |

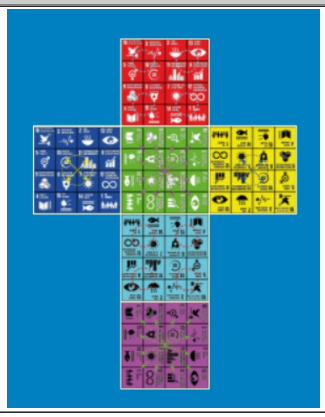
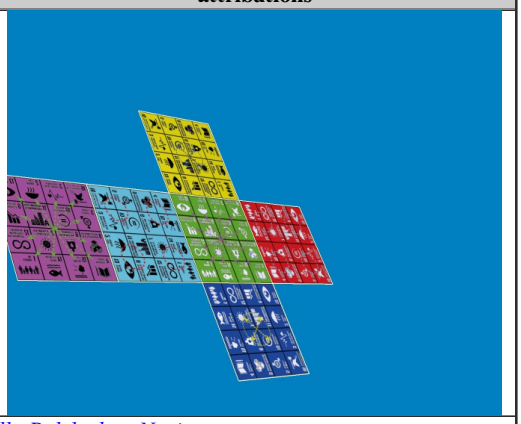
**Sustainable Development Goals**: Such use of colours for a 3x3x3 cubic arrangement of MDGs can of course be applied to a 4x4x4 arrangment of SDGs. How many strategic "colours" are required for sustainability? Using the 6 colours above, a very preliminary experiment resulted in the following. Design choices endeavoured to highlight not only the opposition indicated by use of an opposing (complementary RGB colour) but also the systemic "magic" within the 4x4 pattern of the magic square noted above. Whether or not the pattern of numbers had any significant relation to the numbering of the SDGs, the patterns totalling to 17 within the above table served to highlght that further inquiry might elicit the existence of connectvity which has not seemingly been indicated to date. On that basis it was decided to distinguish:

- on the red and cyan faces of the cube: the 4 circles in the quadrants, each totalling to 34.
- on the green and mauve faces: the 4 sets of relationships denoted by arrows, with each arrow indicating a relationship totalling to 17 (and the symmetrical pairs totalling to 34)
- on the blue and yellow faces: the central square of 4 cells, together totally to 34 (as indicated by a circle), but with arrows indicating relationships each totalling to 17

If only mnemonically, these are the linkages which hold the pattern of 16 SDGs together as an integrative systemic whole -- namely the linkages implied by the 17th SDG: ***Partnerships for the Goals***

The model shown below could of course be improved in various ways. No effort has been made to address the issue of the relevant orientation of the icons on the faces, or of the faces in relation to one another. The lower-left image can of course be used to provide a set of stickers which could be applied to any 4x4x4 Rubik's Revenge Cube (a tabbed version can be used to cut-out and construct the model)

| **Indicative allocation of a 4x4 pattern of 16 SDGs to a 4x4x4 cube**<br>(with indication of systemic patterning in the light of the magic square allocation of SDGs) | |
|---|---|
| Cube presented with all sides | Cube presented with one side rendered transparent |
|  |  |
| Animtions prepared using *Stella Polyhedron Navigator* | |

| **Unfolded cube of 16 SDG attributions** | **Animation of (un)folding of cube of 16 SDG attributions** |
|---|---|
|  |  |
| Images prepared using *Stella Polyhedron Navigator* | |

**Directionality framed by a 3D cube**: It is of course the case that politics has been traditionally defined in terms of the left-right spectrum -- originally derived from the allocation of seats in the French parliamentary assembly. Within this frame there are necessarily only two "sides" to any strategic argument. The possible (or necessary) existence of any other "sides" is considered meaningless in practice -- whatever token reference is made to the diversity of democratic expression.

It is in this sense that a cube offers a pattern of directions consistent with a three-dimensional understanding of reality:

- ***left-right***, as it currently predominates to the effective exclusion of other dimensions. It is on this dimension that extremism is now defined
- ***front-back***, as is evident in the distinction between "front benchers" and "back-benchers" in any assembly, with the latter readily ignored as effectively irrelevant to the real business of government. This dimension is associated with perception of those who are "in" in contrast to those who are "out"
- ***up-down***, as is evident in any hierarchical organization, with those in power at the "top", and those of whom only token account is taken at the "bottom". The dimension is especially evident when politics is somehow associated with religion and its exclusive claim to articulate the values of "heaven" for the benefit of those of lesser distinction enjoined to "believe" (for their own good).

Clearly the three distinct dimensions can be readily collapsed into a single dimension (any of the above) which is then a focus of psychosocial dynamics.

The question is how greater meaning can be given to such multi-dimensionality without (and with) the bias of the special value with which a given direction may be associated -- according to its salience and (in)visibility, as discussed further below.

**6-fold directional framework**: A valuable insight into the variety of perspectives required for sustainability has been offered in a series of studies by Edward de Bono (*Six Frames For Thinking About Information,* 2008). The latter is a generalization of arguments presented in books more widely distributed (*Six Thinking Hats*, 1985); *Six Action Shoes.* 1991; *Six Value Medals*, 2005). It could be claimed that his concern with the MDGs is framed by another study (*New Thinking for the New Millennium,* 1999). The visual depictions of the 6 direction in each case are relevant to more extensive interpretation of Rubik's Cube as a mapping device

**Current example**: The need to take account of an attribution of the above kind is highlighted at the time of writing by the dramatic transformation of the French political scene. Traditional parties have been very significantly marginalized by the public in favour of a new party headed by Emmanuel Macron. An unusual feature of this party is the exclusion from office of those subject to any form of legal investigation, irrespective of the outcome of such proceedings. It aspires to being recognized as a "good guy government" (despite being uniquely distinguished by a majority of women). However one obvious weakness is that it is a minority government brought into power by only 42% of the electorate. The issue then becomes what "colours" and "sides" of public opinion have been effectively "hidden" and how will they manifest over the period of the mandate of the government? Will the Macron regime be found to be essentially "superficial"?

Early criticism has already been voiced in France with regard to the requisite diversity in any democratic process -- especially when faced with the democratic deficit of historic abstention. How is diversity to be integrated into a democratic process when its governance is held to be dependent on a majority with a very particular perspective? As with the debate in other countries, this frames the question as to whether democracy is sustainably governable, as currently conceived (*Ungovernability of Sustainable Global Democracy ? Towards engaging appropriately with time*, 2011).

## Enhancing appreciation of a mapping through a variety of symbols, cultural and otherwise

The symbols used above in the UN iconography of MDGs and SDGs correspond to the manner in which symbols are currently designed for conventional marketing purposes. The advertising industry deploys considerable resources to render "magical" whatever is promoted in a manner which increasingly detracts from the magic otherwise experienced to which people aspire.

The question is whether the cognitive challenge of governance calls for interpretation of value-goals through symbols which engage people otherwise and more fundamentally -- ways which engage their identity in a manner which could indeed be experienced as "magical". The issue is how to relate secular promotion of value-goals with patterns in which people believe as exemplifying and enhancing their sense of identity -- cultural symbols widely appreciated as "magical" attractors.

**Stickers and textures**: Cube users are very familiar with use of stickers to mark cube surfaces with patterns for various purposes -- in addition to that of puzzle-solving as originally framed (see: Normal stickers, Stickers for 3D printed puzzles).

In the virtual reality context envisaged here, the technical equivalent to stickers is "textures". **People could be enabled to switch between symbol systems (and colours) according to preference** -- much as alternative translations are offered on facilities in multilingual environments.

**Languages**: Although the 17 Sustainable Development Goals have been presented through the iconography indicated above, curiously this does not seem to be the case with other languages. Links are not systematically provided to equivalent pages in other languages (as otherwise is often the case) -- even to the official languages of the UN Security Council. The objectives are nevertheless claimed to be such as are necessary to "transform the world".

The icons are however otherwise variously available in French, Chinese, Russian, Arabic, and German, for example. Could a Rubik style cube be understood as providing the integrative functionality of the Rosetta Stone (*Geometry of meaning: an alchemical Rosetta Stone?* 2013; *Systemic Crises as Keys to Systemic Remedies: a metaphorical Rosetta Stone for future strategy?* 2008).

Clearly it would be a simple matter to provide stickers for a Rubrik style cube of SDGs with one language per face -- and even simpler in a virtual reality application. The interesting implication would then be the confrontation of users with other articulations of the same pattern -- especially when non-Latin scripts were used, as with Chinese, Russian and Arabic. The "magical" transformation of the world will not be achieved through English alone -- any more than it was transformed through Latin alone as was the assumption of the Roman Empire and the Holy Roman Empire.

The challenge is reinforced by indications that India will become the most populated country on the planet, followed by China (*World population to hit 9.8 billion by 2050, despite nearly universal lower fertility rates*, *UN News Center*, 21 June 2017). In considering the relevance of Chinese cultural insights, there is a strong case for recognizing the associated subtlety, as helpfully articulated in texts regarding the Literati Tradition (*The Origins of Chinese Philosophical Thinking*; *Analogical Understanding and Translation*; *The Conceptual Scheme of Chinese Philosophical Thinking*).

With respect to analogical understanding, the concluding argument made is:

> Beyond the good intentions of missionaries and sinologists, and the increasing awareness of divesting interpretations of Chinese philosophy of Western preconceptions, the recent archaeological findings challenge the authority of existing translations on Chinese philosophical thinking. To the extent the newly unearthed texts written on silk scrolls and bamboo strips now provide us with a compelling clarity to the over all Chinese cosmology, and thus enable us to understand Chinese philosophical thinking in a way that has not been possible before. Frederick Mote notices that Chinese *history, culture, and people's conceptions of their ideal roles all must be explained in terms of Chinese cosmology, and not -- if we really want to understand Chinese civilization -- by implicit analogy to ours... hence, the records of Chinese culture must be interpreted, and the texts translated and retranslated until our inadvertent uses of historical and cultural analogy are detected, weighed, and if necessary, corrected....*
>
> However, one cannot divest the Western interpretations of Chinese culture of Western cultural analogies simply by translating and retranslating the texts, nor can one do so by using the more abstract and specializing -- no less culturally biased -- language of the professional philosopher. Sarah Allan suggests that one *must begin by exposing the metaphors that underlies the Chinese terminology and imbue it with meaning....* In other words, one begins to understand the conceptual scheme of Chinese

> philosophical thinking more accurately by recognizing the conceptual scheme of Chinese thought, or "the root metaphors" within the socio-cultural contexts from the viewpoints of not only historical and epistemological but also anthropological perspectives, and aesthetic and literary criticism.

**Variations of a Rubik-style cube:** Aside from the increasing complexity associated with the number of squares per face, other kinds of variation have been explored as indicated by Dénes Ferenc (*Shape Cube Modifications*, *Ruwix.com*), including those based on the 3x3x3 pattern of the traditional Rubik's Cube even though they appear quite distinct from it. They are all understood as having the same internal core mechanism, though they have different shapes and solutions. Examples are indicated below.

| **Indication of unusual variants based on Rubik's Cube** (showing the contrast between scrambled versions in the left and central columnsn) | | |
|---|---|---|
| Ghost cube | Mirror cube | Maze and Sudoku cubes |
|  |  |  |
|  |  |  |
| | | |

Especially striking is the contrast between the scrambled and resolved variants in the left and central columns, given that **global governance strategy in relation to sustainable development is essentially faced with a "scrambled" strategic context** -- as recognized in the introductory citations above. The extent and rate of innovation with respect to such devices, and their uptake worldwide, far exceeds that evident in relation to sustainable development goals and their comprehension. E specially noteworthy is the experimentation with shapes other than the cube, notably the cuboctahedron and the dodecahedron.

*** jigsaw puzzle

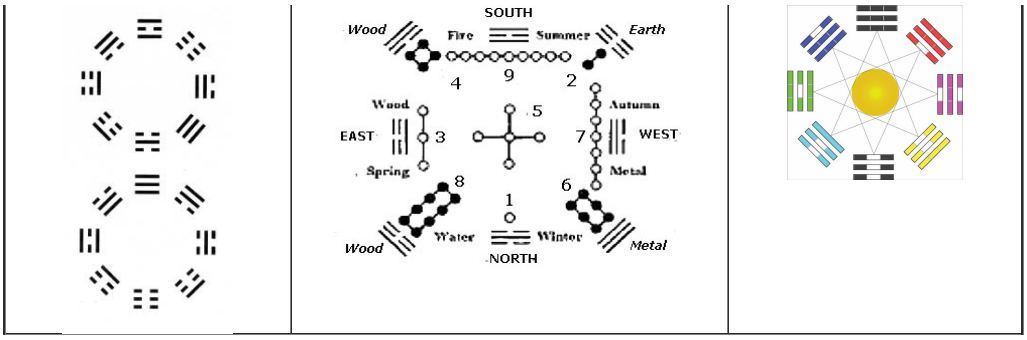
**BaGua pattern of Chinese tradition**: As might be expected, evolution of cube technology has resulted in products variously associated with the *BaGua* pattern mentioned above.

The *BaGua* arrangement has a further advantage in that it is traditionally believed to derive from one of the earliest magic squares -- the 3x3 Lo Shu Square -- which remains a focus of considerable mathematical interest. Histories of development of magic squares make repeated reference to its first recognition in China (Schuyler Cammann, *The Magic Square of Three in Old Chinese Philosophy and Religion*, History of Religions, 1, 1961, pp. 37-80; Frank J. Swetz, *Legacy of the Luoshu: the 4,000 year search for the meaning of the magic square of order three.* A K Peters, 2008).

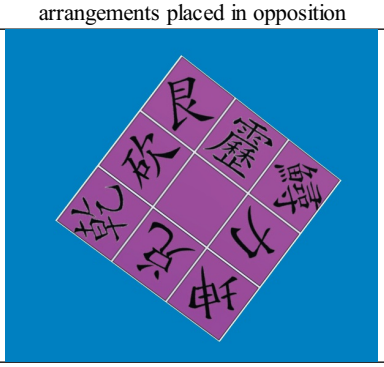
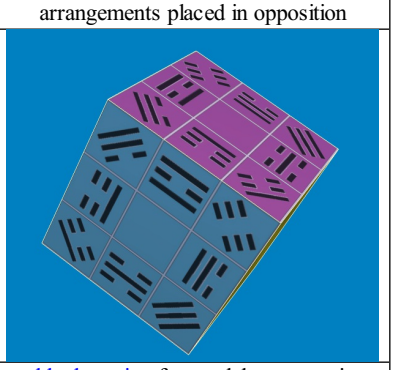
Of particular interest is the transformation of the Lo Shu square into the long-valued *Ba Gua* pattern of 8 trigrams, with its extensive metaphorical associations typically expressed in poetic form, elaborated further within the *I Ching* of 64 hexagrams. The representation of the *BaGua* pattern within that culture necessarily transcends the conventional Western distinction between numbers and letters, given the nature of the script.

The following correspondences feature in an extensive discussion by Quincy Robinson and Paul Martyn-Smith (*Evidence of Modern Physical Knowledge from Asiatic Antiquity: Re-integration: Nine Realms of Middle Earth*, 2015).

| **Correspondences between Lo Shu, Ba Gua and magic square patterns** | | |
|---|---|---|
| ***Classic Ba Gua arrangements*** ("earlier heaven" below, and "later heaven" above). | ***Correspondences*** | ***BaGua configuration of trigrams*** (as an animation) |
| | | |




The clearest example of a model adapted to virtual reality is the Bagua Cube by André Boulouard and Walter Randelshofer (*Chinese Bagua Cube Design*, 2008). The latter includes extensive commentary and many illustrations. The designers have made their design available under a Creative Commons License, as implemented on the left below. No effort has been made to address the issue of the relevant orientation of the icons on the faces, or of the faces in relation to one another.

| Chinese Bagua Cube | | |
|---|---|---|
| **Implementation of design** by André Boulouard and Walter Randelshofer | Fu Xi ("Earlier Heaven") and King Wen ("Later Heaven") arrangements placed in opposition | Fu Xi ("Earlier Heaven") and King Wen ("Later Heaven") arrangements placed in opposition |
|  |  |  |
| tabbed version for model construction | tabbed version for model construction | tabbed version for model construction |
| Images prepared using *Stella Polyhedron Navigator* | | |

A physical model that is far more complex than the original cube is usefully described in one summary (*Konrad's Overview: How to solve the Bagua Cube*; *How to solve the Dayan Bagua Cube*). Neither model endeavours to provide any particular focus on the cognitive implications of the BaGua symbols -- which may (or may not) be of particular significance to its users.

As a game-design issue, of particular interest is whether either the trigram depiction, or the corresponding idiogram, can be presented on a Rubik-style cube face such that a solution to the puzzle-game results in an integrative configuration. Minimally this could be one of the two known "arrangements", but there is the additional possibility that the **designs on each sub-face could then link together to form a more meaningful pattern** (as is the case with some uses of stickers) By contrast, for example, the SDG images above are not designed to offer a larger system perspective when configured appropriately together.

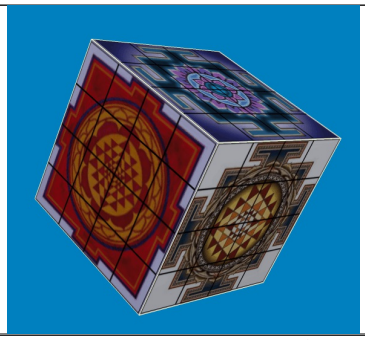
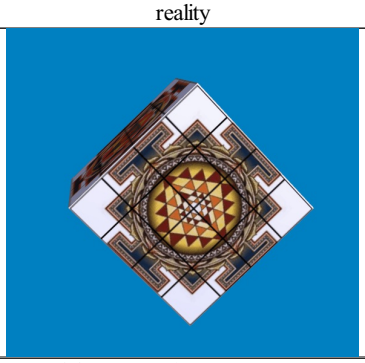
The trigram notation associated with the BaGua pattern has a further advantage in that it offers pointers to how a more comprehensive pattern might be associated with projection onto all faces of a cube. The axes of the cube can be understood as relating to three "directions" which figure in interpretations of that symbolism: North-South, East-West, Heaven-Earth. The basic colours of the cube could be selected to correspond to such an understanding.

To what extent does the articulation development goals by the United Nations constitute an appropriate reflection of correlative thinking -- any more than the pattern articulated in *Agenda 21* (1992)?

**Enneagram**: As might be expected, the 9-fold enneagram has been associated with the 3x3x3 Rubik's Cube (Stan Tenen, *Enneagram - Rubik Cube Hebrew Alphabet Matrix*, Meru Foundation, 1996). However there appear to be no other examples of its representation on a cube.

**Yantras**: The Hindu tradition provides a vast and remarkable collection of systemic patterns known as yantras. Widely appreciated within that culture, they are notably associated with deities representative of value-goals recognizably similar to those articulated as sustainable development goals. Any such pantheon is of course readily deprecated by those alienated by representations and implications valued by non-Western conventions. A particular value of such articulations, however, is the fundamental cognitive significance of the systemic focus. They can be understood as system diagrams with profound psychosocial implications -- at least within that culture. There is a degree of irony to the tendency to recognize speeches in support of conventional value-goals (notably the SDGs) with the expression "mantras" -- when this is the term used to frame appreciation of the deities indicated by the yantras.

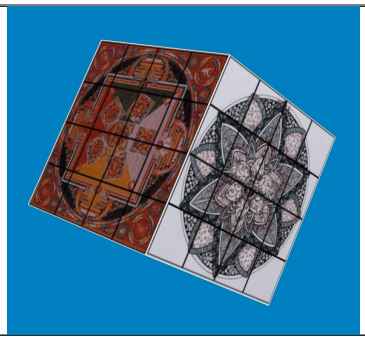
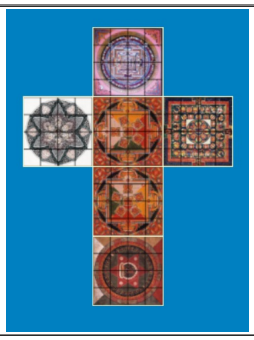
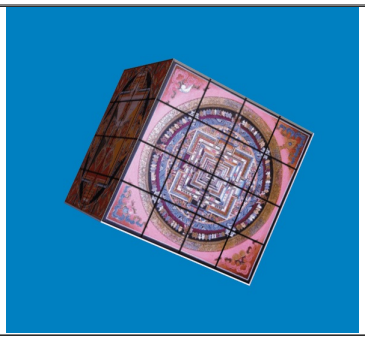
Curiously there seems to be have been little attempt to apply yantras to the surface of a Rubik-style cube, whether as stickers or as virtual reality textures. The following is therefore a suggestive example of how this might be done in the case of the 4x4x4 Rubik's Revenge cube. Rather than apply a yantra to a whole surface, individual yantras (of which there are many) could be applied, optionally and experimentally, to the individual squares on a surface.

| Indication of possibilities of applying yantras to a Rubik Revenge cube surface of 4x4x4 squares | | |
|---|---|---|
| Cube in "solved" configuration | Unfolded cube | Possibility of morphing cube in virtual reality |
|  |  |  |
| Images prepared using *Stella Polyhedron Navigator* (tabbed version for model construction) | | |

Arguably there is a far greater "popular" sense of systemic interconnectivity explored in such yantras individually, whatever the degree of connectivity recognized between them.

**Mandalas**: As with the yantras of the Hindu tradition, Buddhism provides a vast and remarkable collection of systemic patterns known as mandalas. As a spiritual and ritual symbol in both Buddhism and Hinduism and Buddhism, they are understood generically as any diagram, chart or geometric pattern that represents the cosmos metaphysically or symbolically; a microcosm of the universe. Arguably the UN's global governance challenges framed as the SDGs can be understood in such terms. Mandalas are recognized as having a similar significance to the rose windows of Christian churches. Extensive commentary on their value for integrative development has been provided by psychoanalysts following Carl Jung. This has resulted in the articulation of mandalas by those of New Age inspiration -- some of which are included in the images above.

Curiously again, there seems to be have been little attempt to apply mandalas to the surface of a Rubik-style cube, whether as stickers or as virtual reality textures. As above, the following is therefore a suggestive example of how this might be done in the case of the 4x4x4 Rubik's Revenge cube. Rather than apply a mandala to a whole surface, individual mandalas (of which there are many) could be applied, optionally and experimentally, to the individual squares on a surface.

| Indication of possibilities of applying mandalas to a Rubik Revenge cube surface of 4x4x4 squares | | |
|---|---|---|
| | | |
|  |  |  |
| Images prepared using *Stella Polyhedron Navigator* (tabbed version for model construction) | | |

**Viable system model**: Since the pattern of sustainable development goals can be understood as a collective insight into the operation of a viable system, there is a strong case for exploring how to configure the distinguished segments of that model as faces of a cube -- typically in a "scrambled" condition. Of particular relevance is the associated issue of the 4 or more orders of cybernetics which could figure in such a representation -- however those elments might also be "scrambled".

**Platform**: Walter Randelshofer offers a freely downloadable application, CubeTwister, which provides a unique database for cube collections and a maneuver lexicon of value to a wide variety of users. Users can define scripts using their own notation, and can translate scripts from one notation into another. More than 200 interactive 3D models can be explored of: Pocket Cube, Rubik's Cube, Rubik's Barrel, Rubik's Diamond, Cuboctahedron, Rubik's Perpetual Calendar, Rubik's Revenge, Professor's Cube, V-Cube 6 and V-Cube 7. The Virtual Cubes can be twisted, disassembled and even exploded. It includes an extensive collection of more than 4,300 Pretty Patterns for such models. CubeTwister also features a sophisticated cube solver.

## Governance as an ongoing process of puzzle resolution

**Cyclic dynamic**: Use of the term "goal" in both the MDGs and the SDGs reinforces the implication that the relevant strategic processes involve a linear progression towards such a goal -- possibly to be understood as a form of singularity characterized by "sustainability".

This assumption is questionable from a variety of perspectives which could fruitfully inform comprehension of the requisite forms of governance. Obvious examples have been articulated in terms of the adaptive cycle (with its associated resilience). Also of relevance are emerging insights regarding the desirability of a circular economy. The possibility of sustainability is also challenged by widely evident tendencies towards collapse -- as the antithesis of sustainability (*Convergence of 30 Disabling Global Trends: mapping the social climate change engendering a perfect storm*, 2012). A valuable perspective with respect to the nature and allure of the elusive state of

sustainability is offered by David Kahn:

> To benefit from a Rubik's Cube, we must first understand the idea of flow. According to psychology professor Mihaly Csikszentmihalyi, flow is a "state of intense focus and crisp sense of clarity where you forget yourself, lose track of time, and feel like you're part of something larger". It is at these moments of flow state when we're experiencing our greatest engagement, fulfillment, and creativity. (*Rubik's Cubes, Flow State, and Continuous Challenges*, *leadersayswhat*, 5 August 2015)

**Patterns of flow**: With respect to innovation, related insights are offered by Paul Hobcraft (*A Rubik Cube Approach for Innovation*, *Innovation Excellence*, 23 April 2012):

> We need to see innovation as an entire structure where we constantly need to move the parts. We always want to seek alignment, perfect alignment. That dream of achieving all, within that specific aspect of innovation we are attempting, is fully lined up and then we are content. Impossible! What we fail to realize is that the total structure within innovation is always turning; it is always altering its 'face' to adapt to the changes to which we need to adapt. We can never achieve perfect alignment, the makeup of innovation is constantly changing, so what we must do is keep moving the different 'faces' around to meet at best, a temporary alignment. That is the best we can really do....Attempting to align anything needs movement, a sequence of moves. Well if you think about it to achieve a certain innovation 'state' you need to bring a number of aspects or faces to closer alignment but these do need a *planned* sequence of moves in most cases. Relying alone on serendipity just may not work.
>
> Yes the Rubik Cube feels often like innovation, you change one part and it has its consequences on another. When you reflect on all the permutations required in innovation you recognise it is a constant, evolving, ever-changing puzzle. For me the enjoyment of the Rubik Cube is to keep thinking and moving around the different parts.

**Learning metaphors**: Other insights into governance are evident from the well-explored metaphor of yachting -- given the probability of crises and the need to be able to adapt to them. Yachting and similar challenging contexts frame the need for continuing learning (and lifelong education) -- a theme cultivated by UNESCO, but not applied to the UN system as a whole.

Use of Rubik-style cubes embodies the challenges of learning in a variety of ways. As such cube usage can be fruitfully explored as a metaphor in its own right. This merits consideration in the light of the arguments cited above by George Lakoff and Rafael Nuñez (*Where Mathematics Comes From: how the embodied mind brings mathematics into being*, 2001). These notably focus on fundamental cognitive metaphors -- of which those of the directionality characteristic of politics merit careful attention: left-right, up-down, front-back 9as noted above).

There is a curious irony to the manner in which Rubik's Cube constitutes a remarkable form of "container" for the specific arguments of cognitive psychology regarding the so-called "container metaphor" and related cognitive operators (George Lakoff and Mark Johnson, *Metaphors We Live By*, 1980). The irony is especially evident in the manner in which people have "in their hands" a mnemonic learning device with implications for psychosocial development beyond its recognized function as a "brain game".

**Psychosocial implications of cubing algorithms?** Such directions are fundamental to consideration of a Rubik-style cube and the possibilities of its fruitful manipulation towards a desirable solution. As is evident from the many explanations of how to solve such a cube, it is in such terms that a cube is apprehended for that purpose. **The particular gestures, and their sequence, are therefore highly suggestive of a sequence of operations of relevance to achievement of sustainability through appropriate governance**. Especially intriguing is the manner in which there is a need to alternate between the various directions. The cube cannot be solved by repeatedly performing a particular operation -- turning to the right, for example. All six directions figure in any process of solution -- involving a twisting motion recalling the arguments for a paradoxical "cognitive twist" and its potential relation to navigation of the adpative cycle.

Potentially relevant is also unexamined cultural bias favouring particular styles of directionality, as separately explored (*Unquestioned Bias in Governance from Direction of Reading? Political implications of reading from left-to-right, right-to-left, or top-down*, 2016).

Also of interest is the sense in which there is not one single solution process but a multiplicity. Extensive documentation is available on each of them (Marc Anderson *Beginner's Guide to the Rubik'sCube: solution reminder*; *How to Solve a Rubik's Cube (Easy Move Notation*, *WikiHow*; *How to Solve the Rubik's Cube WikiBooks*). These may be distinguished by their difficulty (as with ascending a mountain) and interest. In some cases the explanations may themselves be less than readily comprehensible -- except to the few. This helps to frame the question as to how readly comprehensible is assumed to be the process of progression towards sustainability.

**Metaphorical "translation" of cubing operations -- towards sustainable solutions?** Given the arguments of cognitive psychology regarding the embodied mind, it would be intriguing -- if only as a mnemonic exercise -- to interpret the nature and directionality of the "moves" required for the most direct solution of Rubik's Cube, with the nature of those moves recognized in metaphorical terms as having sociopolitical implications. The provocative implication in democratic terms is the alternation in direction required for resolution -- understood in terms of progressive achievement of sustainability.

There are many presentations of sequences of moves which lend themselves to creative decoding or reinterpretation. A preliminary exercise would clearly be relevant to the pattern of 8 Millennium Development Goals on a Rubik's Cube. The principles could also be applied to the 16 Sustainable Development Goals on a Rubik's Revenge (*Rubik's Revenge Solution Guide*, 1997; *Rubik's Revenge Solution Moves List*). Detailed argument and illustrations lend themselves to interpretation from that perspective, as provided by Jared Weed (*Sub-Optimal Multi-Phase Path Planning: a method for solving Rubik's Revenge*, 2016). Also of relevance is the detailed

argument of Csaba Fogarassy (*The Interpretation of Sustainability Criteria using Game Theory Models: sustainable project development with Rubik's Cube solution*, 2014).

The argument is especially relevant given the role foreseen for artificial intelligence and its potentially implications for decision-making and governance. Clearly the potential patterns of moves have already been decoded or encoded for the solution of Rubik's Cube by such applications (Trevor Mogg, *Robot enters record books for lightning quick Rubik's Cube solution, Digital Trends*, 6 March 2017; *Robot claims fastest Rubik's Cube solving time*, *BBC News*, 9 November 2016). The question is whether the thinking involved can be translated meaningfully into insights into the stages in solving the challenges of governance towards sustainability. Especially interesting is the sense in which the solution of a Rubik's Cube is a multi-stage process in which many steps may well be counter-intuitive -- contrasting with the unfruitful obvious steps which are typically "on the table".

Given the stages in the process of any solution in which the preliminary challenge is to achieve a particular mix of face colours on a face, it would be especially interesting to note the potential significance of such patterns in psychosocial terms.

## Aesthetic coherence essential to attraction of sustainability?

A general case for requisite aesthetics for sustainability is made separately (*Meaningful configuration engendered only by tacit aesthetic entanglement*, 2016).

**17 Symmetries of sustainability?** The argument above was introduced with a discussion of "order in psychosocial space" -- and its implications for strategic design. This focus was the theme of an adaptation of Christopher Alexander's original study (*5-fold Pattern Language*, 1984). "Magic" could then be understood in terms of Alexander's quest as an environmental designer for the core attractor of a "place to be" -- then readily (and commonly) to be described as "magical". In the language of values, and of relevance to governance of sustainable global development, he framed this in terms of a "quality without a name":

> There is a central quality which is the root criterion of life and spirit in a man, a town, a building, or a wilderness. This quality is objective and precise, but it cannot be named., The search, which we make for this quality, in our own lives, is the central search of any person, and the crux of any individual person's story. It is the search for those moments and situations when we are most alive (*The Timeless Way of Building*, 1979)

It is intriguing in relation to 17, that his study (as mentioned above) isolated only 15 "transformations" exemplified by the aesthetics of carpet design -- contrasting with other insights from archtectiural aesthetics. As argued separately, the quest for sustainable governance might indeed be framed in terms of the "magic carpet" metaphor (*Magic Carpets as Psychoactive Systems Diagrams*, 2010).

Architecture makes clear that any major construction of significance requires a preliminary design decision on proportion -- most notably in the case of cathedrals, temples and mosques. Such structures are an embodiment of patterns of numbers, as argued by Keith Critchlow (Keith Critchlow, Islamic Art and Architecture: system of geometric design. 1999). Curiously, with respect to 17, the point is made by the Islamic architecture of the Alhambra in Granada as a focus for the mathematician Marcus du Sautoy as Oxford Professor for Public Understanding of Science. This features in his book on *Symmetry: a journey into the patterns of nature* (2008) and in a TED talk (*Symmetry, reality's riddle*, 29 October 2009). He highlights the 17 symmetries of its decorations, offering the insight that: *The language of group theory gives us the means to prove that 17 -- and no more -- different symmetry groups are possible on a two-dimensional wall*. Taking the Alhambra as an inspiration, together with the variations of Bach, he has offered (with colleagues) a performance of *17 movements, 17 rooms, 17 patterns*.

**Implications of 20 and 37?** The patterns of interaction of the numbers in relation to value-goals can be explored further following the recent importance speculatively attributed to the number 37, especially as the sum of 17 and 20 (*Memetic Analogue to the 20 Amino Acids as vital to Psychosocial Life? Number 37 as indicative of fruitful pathways of transformation?* 2015). Of particular relevance to the above argument is discussion there of *Global strategic significance of 20-fold configurations*, of *Indicative examples of the recognized significance of 20-fold patterning*, and *In quest of number 37 through the pattern of spherical polyhedra*.

**God's number of 20**: With respect to the minimal number of moves required to resolve a scrambled Rubik's Cube, it is then intriguing to take account of mathematical interest in that matter. As noted by *Wikipedia* with respect to *optimal solutions for Rubik's Cube*, There are two common ways to measure the length of a solution to Rubik's Cube. The first is to count the number of quarter turns. The second is to count the number of outer-layer twists, called "ace turns". The maximum number of face turns needed to solve any instance of the Rubik's Cube is 20, and the maximum number of quarter turns is 26.These numbers are also the diameters of the corresponding Cayley graphs of the Rubik's Cube group. (Tomas Rokicki, et al., *The Diameter of the Rubik's Cube Group Is Twenty*, *SIAM Journal on Discrete Mathematics*, 2013; *God's Number is 20*, 14 August 2010). That diameter is known as "God's Number". There are many algorithms to solve scrambled Rubik's Cubes. An algorithm that solves a cube in the minimum number of moves is known as God's algorithm. Ironically it is to that capacity that cubing enthusiasts aspire.

It is very difficult to estimate the God's number of the Rubik's Revenge mainly because the centre piece of it isn't fixed because of which OLL and PLL parities occur. However it is estimated to be between 30 and 33 (Paolo Brolin Echeverria and Joakim Westermark, *Benchmarking Rubik's Revenge Algorithms*, 2013; Tomas Rokicki, *4x4x4 upper bounds: 57 OBTM confirmed; 56 SST and 53 BT calculated*, *Cube Archives*, 3 Mary 2015; Bruce Norskog, *God's algorithm calculations for the 4x4x4 "squares set"*, 4 March 2006; Bruce Norskog, *The 4x4x4 can be solved in 79 moves (STM)*).

**Holy grail of governance?** The 17 value goals suggest that a higher degree of order is required than the fourfold (2x2) magic square highlighted by economics, Metaphorical use of "magic" suggests that the quest for their requiste interplay may as usefully be associated with the ultimate nexus of values associated with the quest for the holy grail (*In Quest of Sustainability as Holy Grail of Global*

*Governance*, 2011; *Interrelating Cognitive Catastrophes in a Grail-chalice Proto-model*, 2006).

**Poetics of policy-making for sustainability?** Relatively little attention is typically accorded to poetry in relation to policy-making, despaite its potential, as argued separately (*Poetry-making and Policy-making: arranging a marriage between Beauty and the Beast*, 1993). Should it be expected that the coherence of a set of 17 sustainable development goals could -- and should -- be meaningfully rendered in poetic form?

The case for the use of poetry to render complexity comprehensible has been well-articulated by biologist/anthropologist Gregory Bateson:

> One reason why poetry is important for finding out about the world is because in poetry a set of relationships get mapped onto a level of diversity in us that we don't ordinarily have access to. We bring it out in poetry. We can give to each other in poetry the access to a set of relationships in the other person and in the world that we are not usually conscious of in ourselves. So we need poetry as knowledge about the world and about ourselves, because of this mapping from complexity to complexity. (Cited by Mary Catherine Bateson, Our Own Metaphor, 1972, pp. 288-9)

Given its declared importance for global civilization, arguably there is a case for poets to be challenged by the need for an epic to achieve widespread appreciation of sustainability and its goals -- as exemplified by the *Mahabharata* or the *Kalevala*. The value for memorability is relatively clear.

In contrast to the articulation of the 17 SDGs or the 8 MDGs by the United Nations, it is notewothy that the Chinese pattern of the *BaGua*, described above, is characterized by extensive poetic implications through the complementary ambiguities of the ideograms through which it is represented. This exentends to their calligraphy -- valued otherwise in that cultural tradition. Less clear evident to other cultures is the "story" which their connectivity implies -- and its extension into the detailed connectivity of the larger story of the *I Ching*.

In relation to the argument above, there is of course the creative possibility of associating poetic stanzas or themes with features of the cube -- the aesthetic challenge being then to form the poem as a whole through resolving the cube from its scrambled condition.

**Sonification?** Could the patterns inherent in the 17 value-goals lend themselves to similar articulation through sonification -- as argued more generally (*A Singable Earth Charter, EU Constitution or Global Ethic?* 2006).

Indications are offered by Michael Staff (*Group theory 101: How to play a Rubik's Cube like a piano*, *TEDEd*, 2 November 2015)

How indeed are people expected to "get their head's around" the cognitive requirements of global complexity -- and then to engage in "sharing values" -- other than through such mnemonic clues to "hands on" sustainable development? This frames the further reflection as to the extent to which one's own sustainable development merits consideration in these terms.

**Coping strategies constrained by intellectual property?** Curiously the appreciation of any works of artists, as envisaged above, is subject to questionable intellectual property constraints. Similar constraints are evident in the history of the development of Rubik's Cube (and its variants) -- framing an additional challenge constraining innovation potentially vital to sustainable development.

This is also the case with the conceptual frameworks and visual aids that can be associated with such devices. All of these tend to be subject to intellectual property constraints and rulings including Rubik's Cube itself, as indicated by a ruling of the Court of Justice of the European Union. (*The Court sets aside the judgment of the General Court and annuls the EUIPO decision which confirmed registration of the shape of the Rubik's Cube as an EU trade mark*. *Press Release*, 122/16, 10 November 2016).

If such devices prove to be vital l to sustainable development worldwide, how is access to them to be enabled rather than frustrated -- inhibiting their further development, as separately argued (*Future Coping Strategies: beyond the constraints of proprietary metaphors*, 1992).

**"Mine" as fundamental to sustainability?** Proprietary constraints suggest that, if only as a mnemonic or aesthetic curiosity, "mine" and its associations play a strange role in relation to the complex of sustainable development goals -- possibly illustrated by the following word-play:

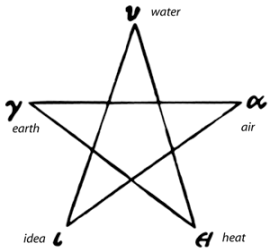
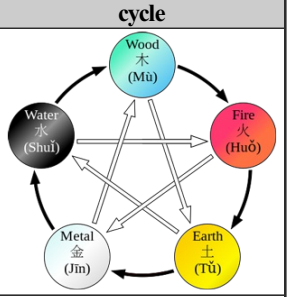
- ***mine***, as exemplifying possession, embodied in issues of intellectual property and copyright. Potentially this extends to use of UN iconogaphy (as above) in relation to the SDGs or the MDGs. It is very evident in relation to the creative development of Rubik's Cube and its variants -- whatever the application enavisaged. It is also evident in the strategic articulation of iconic leaders, possessively formulated as "My Way".
- ***mining***, as exemplified by the mining industry (and mining the seas) in the quest for resources to sustain the lifestyle of humanity. It is hoped to extend this to mining asteroids and planets. In all such cases, forms of encroachment (rendering "mine") are extended through the staking of proprietary claims. This is evident even when natural resources are not explicitly involved, as in the case of territorial claims engendering conflict (*Varieties of Encroachment*, 2004).
- ***mining,*** as exemplified by analogy in the rapidly developing preoccupation with data mining, most notably extending to surveillance issues of security, irrespective of concerns with invasion of privacy. The data collected, whether secretly or otherwise, becomes the jealously guarded possession of the collector.
- ***mine***, as exemplified in the widespread use of land mines and the associated development of improvised explosive devices (IEDs) -- with the concerns raised by minefields, especially when understood metaphorically in relation to psychosocial systems
- ***minding*** the environment, as the caretaker obligations of many sustainable development goals might be understood. This may be extended to reframing the past and selectively envisaging the future (*Minding the Future: thought experiment on presenting new*

*information*, 1980). It can also be understood in the Buddhist sense of mindfulness.

- ***mind***, as the possessive engagement of the mind with the environment, most notably as the effort to "grasp" its complexity in some way and render any explanation "mine" (*Grasping for identity and the challenge of integrative knowledge*, 1997; *Beyond Harassment of Reality and Grasping Future Possibilities*, 1996). The process of "cognitive grasping" can be contrasted with various senses of mindlessness, ranging from mindless indulgence to that articulated by Buddhism (Robert Sharf, *Mindfulness and Mindlessness in early Chan*, *Philosophy East and West.* 64, 2014)

These dimensions play out in the institutional engagement with those sustainable development strategies corresponding to their particular mandates -- and in the proposals made by other bodies and individuals in that regard. Institutions are typically extremely possessive and the quest for sustainability could well be "undermined" by "turf wars".

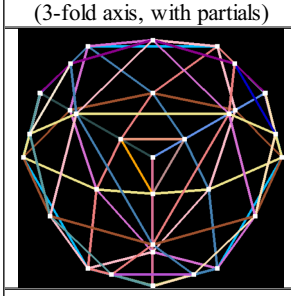
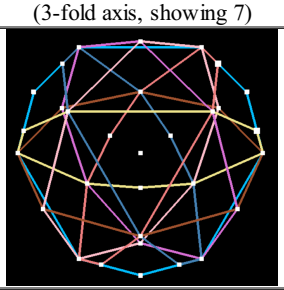
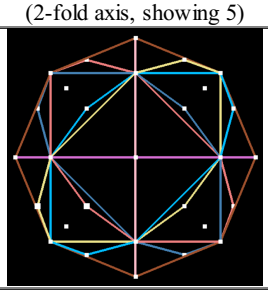
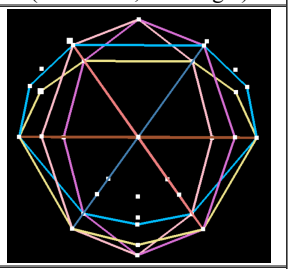
**Fivefold iconic implications of the 17th SDG?** The 17th Sustainable Development Goal is: *Partnership for the Goals*. This is understood as strengthening the means of implementation and revitalizing the global partnership for sustainable development, as framed by the 16 goals discussed above. The fivefold rendering of the iconography (image on left below), whatever the reason for that choice, merits comparison with other traditonal fivefold iconography with related concerns (however implicit) -- and potentially in contrast with those of The Pentagon

| Iconography of 17th UN SDG | Hugieia Pentagram of Pythagoreans | Chinese 5-phase Wu Xing cycle |
|---|---|---|
|   |   |   |
| Reproduced from Global Goals for Sustainable Development | Reproduced from Hygiea entry in *Wikipedia* (G. J. Allman *Greek Geometry From Thales to Euclid*, 1889, p.26) **with labels added** | Adapted from Wu Xing entry in *Wikipedia* **Interaction arrows:** black=generating; white= overcoming |

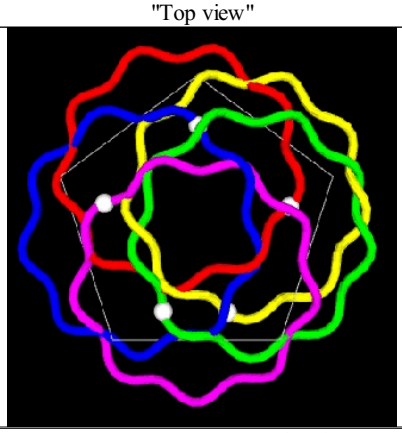
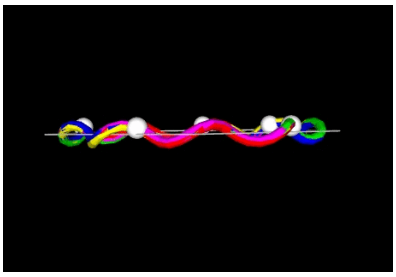
The question of how the fivefold rendering might relate to the 16 goals could be variously explored. The Pythagorean framework (above centre) comes from the origin of the subsequent sense of hygiene which figures in the international iconography relating to health -- with sustainable development perhaps usefully to be understood as a sense of planetary health. The Wu Xing cycle (above right) is fundamental to traditional Chinese thinking on patterns of flow in the environment as generally understood.

These framewoks are dicussed separately in relation to the so-called Pentagramma Mirificum of navigational mathematics (*Global Psychosocial Implication in the Pentagramma Mirificum: clues from spherical geometry to "getting around" and circumnavigating imaginatively*, 2015; *Beyond dispute in 5-dimensional space: Pentagramma Mirificum?* 2015). These drew attention to the curious parallel with respect to health and healing between the pentagonal *Wu Xing* pattern, as a fundamental Chinese concept, and the Pythagorean symbol of the *Hygiea*, also compared separately (*Cycles of enstoning forming mnemonic pentagrams: Hygiea and Wu Xing*, 2012; *Potentially healthy developmental integrity from 5-fold symmetry*, 2012).

The introductory paper explored a mapping of the *Interplay of 17 value-goals -- integrating opposing "hemispheres of the global brain"*. They were rendered as axes across a 4-Frequency tetrahedral geodesic sphere (*Refining the Value of Sustainable Development Goals: in quest of the systemic coherence of global attractors*, 2017). Nine of these axes passed through the centre of that polyhedron. The question could then be how the fivefold set of circles in the UN iconography might relate to that configuration -- if the concern is understanding the systemic integration of those value-goals. One approach is to explore the great circles within that 3D sphere. The images below indicate how the 5 great circles circles of the UN iconography obscure recognition of an additional 2 great circles characteristic of the systemic integrity of the pattern of 17 value-goals mapped into that 3D configuration. This reinforces the argument that a 3D rendering (at least) is necessary for comprehension of systemic integrity.

| **Screen shots of 4-Frequency tetrahedral geodesic sphere** (faces transparent) Illustrating how the perspective chosen renders some of the 7 great circles visible only as a line | | | |
|---|---|---|---|
| Great circles (3-fold axis, with partials) | Great circles only (3-fold axis, showing 7) | Great circles only (2-fold axis, showing 5) | Great circles only (2-fold axis, showing 4) |
|  |  |  |  |
| Images prepared using *Stella Polyhedron Navigator* | | | |

An earlier argument explored the coherence associatd with 5 interlocking cycles of nonagons (*Concordian Mandala as a Symbolic Nexus: insights from dynamics of a pentagonal configuration of nonagons in 3D*, 2016). As illustrated below, a subsequent study provided an animation of 5 such interlocking 9-fold helical cycles (*Visualization in 3D of Dynamics of Toroidal Helical Coils -- in quest of optimum designs for a Concordian Mandala*, 2016).

| **Screen shots of illustrative animation of possible dynamics of 5 coil configuration of partnership among 16 UN Sustainable Development Goals** (Variants: interactive X3D or VRML; videos: MP4 or MOV) | |
|---|---|
| "Top view" | "Side view" |
|  |  |
| Animation prepared using X3D Edit | |

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