¿ Embodying a Way Round Pointlessness ?

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Introduction
This forms part of a more general discussion, where relevant references are located (Way Round Cognitive Ground Zero and Pointlessness: embodying the geometry of fundamental cognitive dynamics, 2012; see alternative table of contents).

The argument there endeavours to reframe the tendency to identify a strategic "way" -- "The Way". Such a way was associated with a "line" and, by implication, with the limitations of linear thinking. The many strategies formulated in this way suggest the need to configure them for greater coherence -- whether as edges of a polygon (in two dimensions) or as sides of a polyhedron (in three dimensions). With the increase in the number of "linear" strategies configured in this way, the configurations approximate progressively to a circle or a sphere (respectively) -- conventionally associated with higher orders of integration.

Mysteries: Curiously this seemingly logical approach fails to address the strange existential mysteries of:

- the questionable impulse to "point-making", with the possibility of engendering a "worldview" -- or be subsumed by such
- the degree to which the context of "point-making" and elaboration of "the way" is characterized by forms of "nothing" -- typically ignored or neglected, most notably in "global" strategies with all-encompassing aspirations
- the manner in which current strategic elaborations of "the way" have engendered "nothing" and the expectation of "nothing" in the future
- the hope associated with "point-making", and the elaboration of a "worldview", of thereby emerging as "number one", thus avoiding being a "no-body"
- the relationship felt between the assumed linearity of "the way", the circular coherence of a desirable outcome, and the nothingness thereby contained or held at bay -- perhaps usefully recognized in aspirations to a "hole in one" in various contexts
- the sense in which seemingly fruitful initiatives tend to get "sucked into a hole" -- a vortex of dysfunctionality or of unbreakable cycles (Web resources on "breaking the cycle", 2002)

There is a sense in which, through the problematic "enclosure of nothing", current understanding of all-encompassing global strategies could be compared to the problematic historical process of the "enclosure of the commons" -- and the consequent "tragedy of the commons".

Naught: In a separate discussion of the potential insights from geometry of higher dimensionality (Global Brane Comprehension Enabling a Higher Dimensional Big Tent? Strategic implication in encompassing nothing and coming to naught, 2011), attention was drawn to the curious relationship between integrative insight and the nature of strategic futility in the light of the contrasting indications offered by Omar Khayyam, the famed Persian polymath (Implicit possibilities of synthesis: Omar Khayyam, 2011):
Of knowledge naught remained I did not know,
Of secrets, scarcely any, high or low;
All day and night for three score and twelve years,
I pondered, just to learn that naught I know.

And if the Wine you drink, the Lip you press,
End in the Nothing all Things end in -- Yes --
Then fancy while Thou art, Thou art but what
Thou shalt be -- Nothing -- Thou shalt not be less.

Rather than a “Way” to engage with any cognitive sense of a “ground zero” and pointlessness, the question explored here is the nature of a “Way Round”. This implies engaging with “nothing” rather than ignoring it and hoping it will go “away”. The approach is encouraged by the increasingly proactive approach of fundamental physics to “nothing” and its potential.

Visual representations of numbers and their cognitive implication

The interplay between the geometrical metaphors, from a strategic and existential perspective, suggests that the argument might be developed further in the light of the manner in which the simplest forms are associated with the visual representation of the simplest numbers and the significance attributed to them. This is most evident in the case of:

- "one" represented by a line, with its association to linear representations of the "One Way" -- the "straight and narrow" (of proper conduct and moral integrity) -- and to various understandings of integrative "unity". Use of a line or linear stroke to represent one has a long tradition.
- "zero" represented by a circle, with its ambiguous associations to a circular target (or integrative outcome), existential futility, knowing "naught", and becoming "nothing" (as indicated by Omar Khayyam). In their use of rod numerals, the Chinese originally described the vacant places as kong, which meant empty. The original form of the Hindu-Arabic numerals did not have the zero symbol to represent the concept and, like rod numerals, had a blank space in its stead. This space was called sunya in India and sifr or as-sifr in Islam, and both words meant empty. (Lam Lay Young and Ang Tian Se. Fleeting Footsteps: tracing the conception of arithmetic and algebra in Ancient China, 1992, pp. 25-27)

There is a curious relationship between the depiction of "one" and a "line". One is readily depicted by a straight line in any process of counting. It is all the more curious in that, as a pole or rod, it is readily associated with "polarization" through which the two ends are implied. A pole also offers a unit of measurement of length -- with "d" (say) being the difference between ends.

A line may of course be curved, then to be described as "non-linear", acquiring special significance as a circle when the ends of a line are joined. Circles may be more readily recognized in nature, as with the eye, or as engendered by dropping a stone in a pond. The relationship between the depictions of "zero" and a "circle" are then equally curious. The sense of "nothing" is then carried by the "emptiness" of the circle.

The curious potential of "nothing" is highlighted by the following conclusion by Terrence Deacon (Incomplete Nature: how mind emerged from matter, 2011), under the heading Nothing Matters:

When Western scholars finally understood how operations involving zero could be woven into the fabric of mathematics, they gained access to unprecedented and powerful new tools for modeling the structure and dynamics of the physical world. By analogy, developing a scientific methodology that enables us to incorporate a fundamental role for the possibilities not actualized - constraints -- in explaining physical events could provide a powerful new tool for precisely analyzing a part of the world that has previously been shrouded in paradox and mystery. The mathematical revolution that followed an understanding of the null quantity in this way may presage a similarly radical expansion of the sciences that are most intimately associated with human existence. (pp. 540-541)

Curiously Recognizing the fundamental remainder engendered by the provocative encounter between one and zero

Cognitive "x-factor" bridging strategically between linear and circular modes: π

It is however the relationship between the straight line and the circle which is especially curious -- when the line is a diameter across the circle (through the centre), or a radius from the centre to the circumference. The length of the circumference of the circle is then described by either of the two forms of the famous relationship:
This relationship has received considerable attention from mathematicians of different cultures down the centuries. Wikipedia records the interest of many, including Archimedes, Liu Hui, Zu Chongzhi, Madhava of Sangamagrama, and Aryabhata.

In the light of the geometrical metaphors explored above, the concern here however is whether the relationship offers further insight into the cognitive implications of "linearity" versus "circularity". The challenge might be exemplified by suggesting that:

- **strategic thinking**, and its associated targeting preoccupations, is typically **linear**, as required by forms of "thinking straight", notably emphasized by widespread use of the "vision" metaphor -- necessarily constrained by "line-of-sight" and the challenge, if only metaphorical, of seeing "round" the globe
- **sustainability**, as a widely represented strategic goal (or target), is arguably more appropriately represented by a **circle**, given the associated importance of feedback cycles, recycling, and the capacity of governance of globalization to navigate the **adaptive cycle**

The adequacy of the "linear" approach to sustainability has been challenged separately (Enhancing Sustainable Development Strategies through Avoidance of Military Metaphors, 1998; Ensuring Strategic Resilience through Haiku Patterns reframing the scope of the "martial arts" in response to strategic threats, 2006).

Linear strategies are challenged by seeing "round", or "in the round", as would seem to be implicit in the requirements for sustainable governance. Curiously, but appropriate to the inadequacies of global governance, this capacity is restricted to ballistics, especially that of intercontinental missiles, as discussed separately (Cognitive Ballistics vs. Derivative Correlation in Memetic Warfare, 2009).

**Enabling cognitive engagement between linear and circular: π as an aesthetic catalyst**

What then might pi fruitfully imply in relation to the metaphorical interpretation of a strategic "line" preoccupied with a focus on elusive sustainability -- characterized by circular processes -- perhaps implied by representation of a circular "target" or "goal"?

**Provocative clues**: The question may be explored in the light of various suggestive "clues", of which the most insightful and comprehensible is possibly sexual interaction and intercourse -- given the widely recognized implications of the above-mentioned visual representation of "one" and "zero", and their interaction.

Considering that such a correspondence is readily framed as inappropriately provocative in conventional terms, this may well be indicative of its value -- given the profoundly fundamental significance of this interaction. It is however appropriate to recall that mathematical speculation on such geometry has been characterized by a "romantic" dimension (Edwin Abbott Abbott, Flatland: a romance of many dimensions, 1884; Norton Juster, The Dot and the Line: a romance in lower mathematics, 1963, with animation). The topological implication has been metaphorically explored separately (Reframing the Dynamics of Engaging with Otherness: triadic correspondences between Topology, Kama Sutra and I Ching, 2011). Arguably the visual resemblance of the lingam and yoni of Indian religions to one and zero respectively is consistent with aspects of this argument -- potentially framing the profound significance associated with such numbers through richer connotations. More provocative, but of potentially equivalent cognitive significance in a period of threats of nuclear strikes, is the resemblance between a missile and the annihilation it ensures.

**Fruitful encounter**: With the "line" as the "diameter" (d) and the circle indicated by its circumference (c) in the above diagram, these are then readily considered as indicative of the male and female organs respectively. The challenge is then how the former "fits" into the latter in fruitful intercourse -- as variously understood. The suggestion is that a **fruitful encounter is enabled by an "x-factor", namely pi**. The linear function of the diameter is then "enhanced" by pi in order to engage appropriately with the circularity of the circumference. Pi could then be understood as holding the complex (non-linear) aesthetic, frequently recognized as a requisite catalyst of the process in practice.

This fanciful schema becomes all the more interesting when the central "point" of any such interaction is considered -- the process by which a "point" is made and any sense of "pointlessness" is transcended. Understood as "conception", in the light of fruitful consumption, the challenge for the "line" is to ensure some form of "encounter" with the invisible "point" at the centre of the "circle" (perhaps mnemonically suggested by the elusive G-spot). Might there be a "sustainability G-spot"?

In the case of a conventional "linear" strategy in quest of sustainability, "zeroing in" on the target is then readily understood as calling for the management "arts" to complement the management "sciences" (Ensuring Strategic Resilience through Haiku Patterns: reframing the scope of the "martial arts" in response to strategic threats, 2006). Reference may be made to preliminary "seduction" as a prelude to forms of strategic implementation described by stronger metaphors and euphemisms. The encounter may nevertheless prove unfruitful, whether or not it results in consumption.

**Boundary sensitivity**: Also of interest is how "boundary" is understood and experienced in the process -- where a sense of "unboundedness" may well be sought and appreciated, whether or not "crossing boundaries" is deprecated. Of relevance, in the light of
the argument above regarding "nothing", is the ambiguity between an all-encompassing sense (engendered in the process) and the challenge of encompassing "nothing" (in the aftermath). For those intimately engaged therein, strategic initiative may well involve an analogue to post-coital distress.

This ambiguity may be recognized in the contrast between identification with the full flow of strategic implementation and the experience of the strategic aftermath -- at the end of the life of the initiative, whether or not there has been any "sunset provision", or provision for recognition of "negative impacts". Strategies may well be promoted and implemented as though they are eternal -- echoing an experience characteristic of sexual relationships. There are echoes of this where either is variously associated with a "divine right".

Appropriate "marriage": The merit of the schema is that it frames the "x-factor" in the challenge of both ensuring "goodness of fit" and of achieving the degree of mutual entrainment necessary for linear thinking ("thinking straight") to mesh sustainably with circularity and ensure "fulfillment" -- whether or not this is understood in contractual terms. The implications of the metaphor may be further enhanced by consideration of the irrational nature of pi and the approaches to its approximation.

Pi could also be understood in terms of the nature of the "secret" to the "bond" in any successful "marriage", whether of individuals, groups or otherwise incommensurable principles -- or even in relation to what Carl Jung explored as the divine marriage (Mysterium Conjunctionis, 1955). This has notably been associated with the participation mystique (Remo F. Roth, The Archetype of the Holy Wedding in Alchemy and in the Unconscious of Modern Man, 2005).

The mysterious relation between zero and one is effectively echoed by the ritual placement of a ring on a finger in celebration of the wedding contract. Curiously it is through the "conjoining" of one and zero that a new cycle is enabled in the number system.

Imaginative engagement: As framed within this metaphorical context, the significance of pi is necessarily a matter open to the "imagination" -- one aspect of the aesthetic nature of the catalysis required to bridge between "linear" and "circular". As such it also has a quality of "dream". A nicely appropriate coincidence, of mnemonic value to the above argument, is that the Chinese classic on poetry, Shih Ching, distinguishes six genres of poetry, of which one, identified with metaphor, is named pi -- as noted by Kuang-ming Wu (On Metaphoring: a cultural hermeneutic, 2001, p. 563-4).

Is there a case for exploring a "Kama Sutra" of global sustainability as suggested above-- if only as a provocative mnemonic aid? More provocative is the strange historical coincidence between "sustainability" and the worldwide preoccupation with aphrodisiacs, mostly notably viagra. This emphasis in fact contrasts with the primary -- but lesser known -- focus of the Kama Sutra on aesthetics.

Strategic implication of $\pi$ through polygonal approximation to a circle

A "mystery", for those who have reflected on the nature of the relationship between line and circle, is why division of the circumference by the diameter is not an integer, namely exactly three -- as an appropriate "universal constant". Alternatively the question is why there is a fractional part, following the integer part, namely the part to the left of the decimal separator (conventionally indicated by a point).

Much attention has been given to the calculation of the fractional part to many digits, notably to determine whether there was any form of closure.

This situation then raises the question of the significance of the fractional "remainder", given that the diameter (or radius) can never be an integral good "fit" in relation to the circumference. Linearity can never satisfy and fulfil the circularity required by globality.

Curiously, in the light of the argument in the main paper, the earliest methods of calculating pi -- by Archimedes (in Greece) and Liu Hui (in China) -- have been to provide approximations through the use of polygons. As noted in the Wikipedia entry on pi, with respect to Liu Hui, his algorithm for pi was one of his most important contributions to ancient Chinese mathematics. It was based on calculation of the area of an N-sided polygon, in contrast to the Archimedean algorithm based on polygon circumference.

Archimedes used a circumscribed 96-gon to obtain an upper limit $p < 22 / 7 = 3.142857$, and an inscribed 96-gon to obtain the lower limit $223 / 71 = 3.140845$. Liu Hui was able to obtain both his upper limit $3.142704$ and lower limit $3.141024$ with only an inscribed 96-gon. Furthermore, both the Liu Hui limits were tighter than Archimedes': $3.140845 < 3.141024 < p < 3.142704 < 3.142857$. With this method Zu Chongzhi subsequently obtained the result: $3.1415926 < p < 3.1415927$, which held the world record for the most accurate value of pi for 1200 years. (Robert K. G. Temple, The Genius of China: 3,000 Years of Science, Discovery and Invention. Prion, 1998, pp. 144-145).

Fig. 1: Progressive approximation of increasingly complex regular polygons to a circumscribed circle

as indicated by area and by perimeter for polygons up to 108 edges

(derived from Rutgers Regular n-gon Calculator and The Regular Polygons Calculator)
Note that it is only from the (plus or minus) 12-edged polygons that approximation to a circumscribed circle becomes significant -- at over 90%. This reinforces the case for 12-membered "round tables" and juries as offering good approximation to "well-rounded" judgement. The more general strategic implications are discussed separately (Eliciting a 12-fold Pattern of Generic Operational Insights, 2011; Enabling a 12-fold Pattern of Systemic Dialogue for Governance, 2011; Checklist of 12-fold Principles, Plans, Symbols and Concepts, 2011).

Of relevance in terms of a metaphorical perspective, a strategic "line" may assume some sense of where the (invisible) "point" is located but be unable to "pin it down", even when passing "through it". As a zero-dimensional entity, the point cannot be "grasped" or "got" -- conceptually or otherwise. Furthermore a single "line" does not effectively engage with the "nothing" by which it is surrounded within the encompassing circle. However, the more lines there are -- configured as a symmetrical polygon -- the more distant such lines are from the central "point" of the polygon (through which they no longer pass), although together they encompass "nothing" (to a degree) in terms of a "virtual" centre.

This argument suggests that a container for "nothing" is required in order to encompass "everything" without falling into the cognitive trap of endeavouring to "grasp" the point of it. A curious implication in the case of the fractional part of pi, despite the number of digits identified, is the manner in which it fails to "converge". This confirms the sense in which perfect engagement with sustainability -- the perfect "marriage" -- is felt to be impossible to achieve. Appropriately a controversial effort has been made to reframe the quest of physicists for a Theory of Everything in terms of "nothing" (Russell Standish, Theory of Nothing, 2006).

Cognitive implication of polygon circumcircle and incircle in strategic implementation

As noted above, the earliest approaches to pi in China and Greece were in terms of of the circumcircle or incircle of a polygon. There appears to be little trace of any cognitive implication attached to these geometric devices -- although both figure in various cultures, in symbolic representations based on a polygon.

**Circumscribed circle**: With respect to the argument here, the circumcircle is readily understood as the larger circular dynamic of sustainability with which "principles" and "values" may be variously associated, if only by implication -- as distinct "points" on that circle. The encompassing circumcircle can therefore be understood as a form of (distant) boundary to human aspiration -- the integrative fulfillment in principle (as an ideal) of which claims are made, notably in political rhetoric. Individual linear strategies traverse that space with forms of implementation (possibly focused and reinforced by "tunnel vision") towards fulfillment of such values.

The set of strategies, understood as an effort to encompass sustainable governance, may then be usefully represented as a polygon within the circle -- whose strategic coherence is best implied by the symmetry of the polygon. Such strategies may then be readily associated with the "strategic pillars", typically considered as fundamental to many international initiatives. These are however rarely, if ever, understood to be configured as a polygon (Coherent Value Frameworks: pillar-ization, polarization and polyhedral frames of reference, 2008).

**Inscribed circle**: The incircle might then be appropriately understood to imply the degree of integrative fulfillment in practice, as a consequence of what the linear strategies are together able tangentially to "contain" and "ring fence". The incircle of course echoes the form of the circumcircle and rhetoric may readily conflate the two, implying that the achievements suggested by the incircle are those promised in terms of the circumcircle. Curiously the world is tolerant of so-called puffery with respect to such claims, however much they are an imaginative stretch (as with the sexual analogue).

Cognitively this confusion necessarily applies to any form of "marriage", with the circumcircle representing the fulfillment hoped for and acclaimed, whilst the incircle represents the reality in practice -- "at home" and "on the ground". The difference between what the latter encompasses, and the aspirations associated with the former, are a primary source of despair. It is in relation to the incircle that the sense of being "fenced in" arises.

| Fig. 2: Comparative degrees of approximation of polygons to a circumscribed circle in contrasting cases including inscribed circles to suggest degrees of "dissonance" between aspirations to circularity and that effectively contained |
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| Triangle: minimum approximation, maximum "dissonance" (3 edges) | Pentagon: medium approximation, medium "dissonance" (5 edges) | Dodecagon: high approximation, minimum "dissonance" (12 edges) |
Nothingness and the containment of globality: The images of Fig. 2 further clarify the issue of the limited ability of some polygons to "contain" the sense of globality to which their linear component strategies may purportedly aspire. In this sense the portion of the circumcircle (which the polygon fails to "occupy") may be experienced as a form of "nothingness" -- however great the unrealized potential, which may be (confusedly) "felt" to be associated with it. The area of the incircle offers a different quality of nothingness -- perhaps as implied by the notorious "known unknowns" of Donald Rumsfeld, as separately discussed (Unknown Undoing; challenge of incomprehensibility of systemic neglect, 2008).

The inscribed circle, tangentially framed by the edges of the polygon, is indicative of the form of globality held to be effectively "contained" in practice. In this sense a single linear strategy (as with two) is unable to ensure "containment" in practice -- exemplifying the challenge of engaging meaningfully with "nothing". The image in Fig. 2 of the 12-edged dodecagon is a valuable indication of the larger number of differently oriented strategic elements required to approximate to globality with a minimum degree of "dissonance".

Expressed otherwise, dynamically and provocatively, the less the strategic capacity and scope associated with any single initiative, the greater the number of successive strategic "thrusts" required to achieve global "fulfillment" -- then dependent on the art of ensuring the collective coherence of the dynamic of the alternation between them -- perhaps usefully understood as a standing wave. Framed in this way, and considering the UN's Millennium Development Goals as an example of a set of eight individual strategic "thrusts", the question is whether these are configured together sufficiently coherently to indicate and encompass a sustainable "global" goal -- and how any such aspiration might then contrast with what they tangentially contain in practice.

"Mini-incircles": The geometry of the polygon also offers other insights as illustrated in Fig. 3. Especially interesting is the sense in which multiple "mini-incircles" become apparent under certain circumstances as "mini-fulfillments", achieved tangentially in relation to a smaller number of strategies (a subset) than that held to be relevant to the larger sense of sustainability.

Such fulfillment to a lower degree in these terms might well be understood as a consequence of "subunderstanding" in the light of the argument of Magoroh Maruyama (Polyocular Vision or Subunderstanding? Organization Studies, 25, 2004) and appropriately considered as relevant to agriculture, as admirably described by Egon Noe, et al (A semiotic polyocular framework for multidisciplinary research in relation to multifunctional farming and rural development, 2005).

Identification of the zones of potential mini-fulfillment may be assisted by some online polygon calculators. These may identify features such as the number of:

- **diagonals**: the proper diagonals in a regular polygon, namely excluding the \( n \) sides).
- **intersections**: the intersections (including the \( n \) vertices) of all the diagonals of a regular polygon
- **regions**: the regions into which the diagonals cut the interior of the polygon
- **segments**: the segments (including the \( n \) sides of the polygon) that the diagonals cut each other into

These may variously identify spaces within which "mini-incircles" are tangentially defined. Such spaces may also be engendered by strategies which are not configured into any symmetric polygonal pattern, or take any polygonal form. This is probably the most typical case, as would become apparent if efforts could be successfully made to map strategies, as discussed with respect to the Global Strategies Project (Strategic Ecosystem: feedback loops and dependent co-arising, 1995).

| Fig. 3: Illustration of multiple strategic "mini-incircles" in the case of the hexagonal polygon in contrast with the case of the incircle and circumcircle |
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Implication of \( \pi \) through approximation of strategic polyhedra to a sphere of "globality"

As a geometrical metaphor (as argued in the main paper), the sphere is readily held to be of more fundamental integrative significance than the circle. The arguments above relating to \( \pi \) apply equally in relation to the sphere. Cognitively this necessarily offers a more encompassing "global" sense of fulfillment. The form epitomizes, to a greater degree, the strategic challenge of the "vision" of a multidimensional "holy grail" of global governance, as separately discussed (In Quest of Sustainability as Holy Grail of Global Governance, 2011).
As described above, one or more linear strategies are effectively challenged to engage and encompass the circularity of sustainability. In the case of the sphere, however, strategies are understood to be variously configured as a number distinct polygons. These together form the sides or surfaces of a polyhedron -- potentially symmetrically configured. Corresponding to the progressive approximation of a polygon to a circle, with the increase in the number of its edges (clarified by Fig. 2 and Fig. 3), the set of sides approximates to a surrounding sphere as the number of sides increases (as shown in Fig. 4). As with the circle, pi is necessarily the "x-factor" through which the "superficiency" of the variously configured polygons is reconciled -- to a degree -- with the "globality" by which sustainability is held to be characterized.

Approximating globality: As with the case of the approximation to a circle, the question is then how many "strategic sides" fruitfully approximate to a sphere? Using the term "sides", of particular relevance to coherent global governance is the extent to which these polyhedral sides may be felt to be explicitly associated with mutually challenging political or ideological "sides". In this language, as the geometry illustrates, the smaller the number of sides (configured within the circumscribed sphere) the lower the ability to engage effectively with the globality implied by that sphere. There is then a case for exploring the number of sides -- characteristic of different polyhedra -- which could be held to approximate more "adequately" to the sphere, as was done in the case of the circle (Towards Polyhedral Global Governance: complexifying oversimplistic strategic metaphors, 2008).

As in the comparison of the polygons (in Fig. 1), Fig. 4 shows the greater relative capacity of the dodecahedron and icosahedron to approximate to the sphere -- but only to the extent of 65-85%, depending on whether the comparison is made in terms of surface area or volume. It is only with the most complex of the semi-regular polyhedra, the 92-sided snub dodecahedron, that in excess of 90% approximation is achieved to globality.

Recovering functional insights from myth? This offers a degree of justification for the significance attributed to the association between the dodecahedron and the 12 Olympian Gods of Greek mythology, known as the Dodekathen -- adapted as the Dii Consentes (also Dii Complices) in the pantheon of Ancient Rome, with its twelve major deities, six gods and six goddesses. This association might be understood systemically as an effort to reconcile the functional perspectives (or "sides") associated with each deity in order to give form to an elusive larger "global" system -- which otherwise readily escapes comprehension.

A similar argument has been made for other 12-fold patterns of articulation, as noted above (Eliciting a 12-fold Pattern of Generic Operational Insights, 2011). Particularly intriguing is the functional significance of the "relationships" between the gods as traditionally highlighted in numerous mythological tales, especially as indicative of requisite patterns of dialogue in governance (Enabling a 12-fold Pattern of Systemic Dialogue for Governance, 2011). Given the viability of these cultures and their governance over centuries -- exceeding those of current times -- a degree of prudence is appropriate in any depreciation of such implications out of hand.

It is curious, by contrast, that since those times little of functional significance is recorded regarding the individual relationships between, for example, the "12 Knights of the Round Table", the "12 Imams", or the "12 Apostles" (even at the Last Supper). This reinforces the case for "mining" myth for systemic insights, as argued by Susantha Goonatilake (Toward a Global Science: mining civilizational knowledge, 1999).

Requisite variety for global sustainability: Especially striking in the case of the graph in Fig. 4 is the extent to which the regular polyhedra (with fewer sides), including the dodecahedron, only partially approximate to the sphere -- to the extent that this is understood to imply a capacity to encompass globality. The greater capacity to do so, of the more complex semi-regular polyhedra, could be understood as indicating that a greater variety of functional "perspectives" and "viewpoints" is required to articulate that globality.
adequately -- a greater requisite complexity in cybernetic terms, in the light of the Law of Requisite Variety. What might be those contrasting insights? Does "semi-regular" suggest integration of "alternative" perspectives to complement "regular thinking"?

**Patterning variety:** Any such requirement would then raise questions of how these characteristic are to be "re-cognized" in practice and of how they might be fruitfully and comprehensibly configured together -- especially when each such contrasting perspective typically gives rise to a "side" in political and ideological dispute. Some examples have been explored separately (Polyhedral Pattern Language: software facilitation of emergence, representation and transformation of psycho-social organization, 2008). Contiguous "sides" may of course be understood as potentially forming coalitions with sub-global preoccupations.

In approximating most closely to a sphere, the 92-sided snub dodecahedron is an example of such a configuration. It is oddly reminiscent of the 92 chemical elements found in nature and of the manner in which they can be configured in the periodic table of chemical elements. The possibility of exploiting this pattern to elicit functionally significant psychosocial relationships has been discussed separately (Tuning a Periodic Table of Religions, Epistemologies and Spirituality: including the sciences and other belief systems, 2007).

Given the fundamental importance attached to the "vision" metaphor, another approach is to explore the movement of light within facetted crystals as polyhedra -- the precious stones and jewels so fundamentally associated with value and insight. In order to maximize their effect in channelling light, these are cut into patterns of facets -- extending the applicability of the vision metaphor in the light of the optical possibilities of crystals, as separately discussed (Patterning Archetypal Templates of Emergent Order: implications of diamond faceting for enlightening dialogue, 2002).

**Cognitive implication of polyhedron circumsphere, insphere and midsphere in strategic implementation**

In extending the arguments above regarding the capacity of a polygon to "contain" globality (as illustrated by Fig. 2), corresponding arguments to the relationship between the circumscribed circle and the inscribed circle can be made regarding the relationship between the circumscribed sphere of a polyhedron and its inscribed sphere -- to which the sides of the polyhedron are tangential.

In a period of intense focus on the strategies appropriate to "globalization" and its governance, there are intriguing associations to be explored between these two spheres in relation to the ideal of sustainable global governance and that achieved in practice. As in the case of the circles in the polygon case, the larger encompassing sphere is suggestive of the ideals of global sustainability -- with its association to the planet as a whole. The inscribed sphere is suggestive of what is achieved in the practice of governance -- with a notable discrepancy between them. In the light of the rhetoric of public relations, association of the circumsphere with strategic ideals may even be fruitfully caricatured as a (trial) "balloon", as discussed separately (Globallooning -- Strategic Inflation of Expectations and Inconsequential Drift, 2009).

As in the polygon case, both spheres are indicative of the challenge of engaging with "nothing" -- dependent on "sides" in the polyhedral case, so evidently problematic in any effort to achieve strategic coherence in reconciling conflicting political "sides" in a balanced manner. Again, as indicated by Fig. 6b, the 92-sided snub dodecahedron (with both triangular and pentagonal sides) approximates more closely to the circumsphere than the 12-sided dodecahedron (with only pentagonal sides).
Intersphere: Of further interest in the polyhedral case, is the existence of a midsphere (or intersphere). This is tangent to every edge of the polyhedron, touching the edge at a single point. The midsphere is so-called because it is between the inscribed sphere (which is tangent to every face of a polyhedron) and the circumscribed sphere (which touches every vertex), as indicated by Fig. 5 and Fig. 6 respectively.

It is appropriate to note that the theologically inspired depictions of body, soul and spirit often make use of concentric circles to suggest their relationship -- by thereby implying concentric spheres. The argument here has associated aspirations to "globality" (and the highest human values) with the circumscribed sphere. The strategic reality, as implemented and experienced "on the ground", has been associated with the inscribed sphere. Cognitively it could be argued that the integrative sense of "globality" is however "within" rather than "without" -- in greater accord with the theological attribution. Cognitive alternation between these senses -- "without" or "within" -- therefore also merits reflection especially in the light of the more paradoxical geometric possibilities of higher dimensionality, such as the Klein bottle (Intercourse with Globality through Enacting a Klein bottle: cognitive implication in a polysensorial "lens", 2009).

As an intermediary, the intersphere invites other interpretations of strategic and cognitive significance. It was suggested above that in strategic rhetoric the reality of implementation associated with the insphere was readily conflated -- for purposes of puffery -- with the hopes, aspirations and political promises associated with the circumsphere. Similar conflation is evident in the distinctions made between "spirit" and "soul", which may well be used interchangeably -- although "soul" may be associated with a more worldly and less transcendent aspect of a person. Notions of "spirit" are of course typically a feature of religious "promises".

Such potential confusion, whether inadvertent or deliberate, raises the question as to whether there are three understandings of global sustainability to be distinguished -- associated in some way with the fulfillment of individual, communal, and cultural identity, for example?

"Mini-inspheres": As with "mini-incircles" in the case of polygons (see Fig. 3), the geometry within polyhedra creates "sub-spaces"
within which "sub-spheres" indicative of more limited understandings of globality and sustainability may become evident and be upheld as fulfilling.

"Mini-cycles": Given the above discussion of "mini-incircles", also of interest is the manner in which the polygonal sides of a polyhedron each create a space for the incircles discussed in Fig. 2. These are variously depicted in Fig. 8 in which the "circles" are treated as "cycles" with directionality. Whilst "conflict" in any implied meshing between these cycles can be "avoided" in the two-dimensional representation, folding this into the global form of the polyhedron makes such conflict between contiguous cycles apparent -- if it is assumed that there is any systemic exchange between them necessary for global sustainability.

**Fig. 8: Indication of the possibility of polygonal incircles on the sides of a polyhedron**
as indicated in the case of a dodecahedron and its representation as an unfolded net
(Note the directional "conflict" between cycles associated with such circles)
(images generated using *Stella Polyhedron Navigator*)

What are the interlocking cycles necessary to sustain integrative globality? The question has been variously explored separately (*Configuring interlocking cycles*, 1994; *Interlocking cycles enabling psychoper operation*, 2011; *Omnitriangulation: interlocking cycles*, 1983).

**Seven circles theorem?** Of potential relevance to the argument above is the *seven circles theorem*, as illustrated by the following image.

Fig. 9 illustrates a theorem about a certain arrangement of seven circles in the Euclidean plane (see Stanley Rabinowitz, *The Seven Circles Theorem*). Specifically, as noted by *Wikipedia*, given a chain of six circles all tangent to a seventh circle and each tangent to its two neighbours, the three lines drawn between the six points of tangency with the seventh circle all pass through the same point. Though elementary in nature, this theorem was not discovered until 1974. Other illustrations are offered by the *Wolfram MathWorld* description of the theorem. An interactive applet is provided on the Cut-the-Knot site.

This offers a sense of how difficult it may be to determine the "point" within a global context -- in which various "circles of opinion" seek to prevail.

The pattern in two dimensions suggests that a similar pattern, involving tangential planes, may exist in three dimensions -- which may have even more significant implications for an understanding of "globalization" and the sub-understanding with which it may be associated. There is indeed a mathematical literature regarding matters relating to "seven spheres" (D. Montgomery and C. T. Yang, *Differentiable Actions on Homotopy Seven Spheres*, American Mathematical Society, 1966). A related literature regarding *seven dimensional spheres* has been fundamental to topology. Curiously "seven spheres" have been a long-standing preoccupation of the theological speculation of various religions.

Also of relevance is the extensively studied question of *sphere packing* and the manner in which the (semi) regular polyhedra pack together, as indicated in the following image (Fig. 10).

**Fig. 10: Arrangement of the 12 Archimedean polyhedra in their most regular pattern, a cuboctahedron, around a truncated tetrahedron**
(from Keith Critchlow, *Order in Space*, 1969, p. 39). Arrows indicate the succession of truncations from 1 to 6 in each case.

[Disabled: Clicking on a polyhedron links to a spinning image]
A spherical framework raises the possibility of a more appropriation distributing of categories to enable a genuinely sustainable "global" governance, as explored separately (Spherical Configuration of Categories to Reflect Systemic Patterns of Environmental Checks and Balances, 1994; Spherical Accounting: using geometry to embody developmental integrity, 2004; Spherical configuration of interlocking roundtables: Internet enhancement of global self-organization through patterns of dialogue, 1998; Spherical Representation of Icosidodecahedral Net of Strategies, 1995).

Enabling a reconciliation between one and nothing: π and the mysterious Euler identity

The so-called Euler identity (or Euler equation) has been named as the "most beautiful theorem in mathematics" and has tied in a nomination by mathematicians for the "greatest equation ever" (Robert P. Crease, The greatest equations ever, PhysicsWeb, October 2004). It is presented as follows:

<table>
<thead>
<tr>
<th>Euler identity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^{i \pi} + 1 = 0$</td>
</tr>
</tbody>
</table>

- $e$ is Euler's number, the base of natural logarithms,
- $\pi$ is the ratio of the circumference of a circle to its diameter, as discussed above.
- $i$ is an imaginary number defined by its property $i^2 = -1$.

This is consistent with 3 variants: $i^0 = 1$, $i^1 = i$, $i^3 = -i$

As noted by Wikipedia, its mathematical beauty is associated with its use of the three basic arithmetic operations only once: addition, multiplication, and exponentiation. It also links five fundamental mathematical constants (Five constants tie together multiple branches of mathematics, 2008):

- the number 0, the additive identity.
- the number 1, the multiplicative identity.
- the number $\pi$, which is ubiquitous in trigonometry, the geometry of Euclidean space, and analytical mathematics
- the number $e$, the base of natural logarithms, which occurs widely in mathematical and scientific analysis ($e = 2.718281828...$).
  - Both $\pi$ and $e$ are transcendental numbers.
- the number $i$, the imaginary unit of the complex numbers, a field of numbers that contains the roots of all polynomials (that are not constants), and whose study leads to deeper insights into many areas of algebra and calculus, such as integration in calculus.

In a context referring to the geometry of polyhedra, the Euler identity should not be confused with the Euler characteristic, originally defined for surfaces of polyhedra used to prove various theorems about them, including the classification of the Platonic solids:

$$\chi = V - E + F = 2.$$  

where $V$, $E$, and $F$ are respectively the numbers of vertices (corners), edges and faces in the given polyhedron. The identity should also not be confused with the Euler formula, named by the physicist Richard Feynman as "our jewel" and "one of the most remarkable, almost astounding, formulas in all of mathematics":

$$e^{ix} = \cos x + i \sin x$$

However, given that Euler's identity is a special case of Euler's formula, arguments (including metaphors) to enable understanding of the formula are instructive with respect to the identity. One example is its graphical representation by a circle inscribed by a unit radius passing through all possible angles. The most remarkable is that of Kalid Azad (Intuitive Understanding of Euler's Formula, 2010) for whom Euler's formula describes two equivalent ways to move in a circle. With respect to the identity, others suggest that it is a mathematical tool for converting between polar coordinates and rectangular coordinates on the complex plane.

In one form or another, the Euler equation provides a key to understanding the transformation between linear and circular cognitive modes -- an aspect of the challenge posed by the argument above.

Nothing and unity: As framed above, both the psychosocial and strategic challenges are with respect to achieving:
• a more fruitful relationship between "nothing" (so central to the problematic experience of existential despair) and
• the aspiration for a sense of integrative "unity" (so central to the questionable aspiration to recognition as being "number one").

Strangely, and probably appropriate to the comprehension challenge however, both have ambiguous connotations. Thus "nothing" also offers echoes of a sense of integrative globality and fulfillment -- if only through sexual interpretation of its representation. Similarly the "unity" implied by being "number one" calls for a degree of integration, typically problematic in practice -- also readily recognized in the challenges of sexual impotence.

The argument has focused on geometrical metaphors for the interpretation of one and zero, through line and circle, using \( \pi \). The Euler identity offers another way of exploring the relationship between the cognitive implications of linear and circular modalities -- involving \( \pi \) otherwise. Through visualization, the use of geometrical metaphors rendered relatively comprehensible the specific role of \( \pi \) in the "aesthetic" reconciliation between line and circle. However the embedding of \( \pi \) in this strange "identity" would appear to pose fundamental problems of comprehension of a quite different order.

<table>
<thead>
<tr>
<th>Variant presentations</th>
<th>Possible implications</th>
</tr>
</thead>
<tbody>
<tr>
<td>( e^{i\pi} + 1 = 0 )</td>
<td>Suggests that a strange &quot;x-factor&quot; is required to supplement any unitary modality (especially of a linear strategic form) in order to engage effectively and sustainably with globality</td>
</tr>
<tr>
<td>( e^{i\pi} = 0 - 1 )</td>
<td>Suggests that the absence of the &quot;x-factor&quot; is to be recognized through the removal of a unitary modality from globally encompassing understanding. Metaphorically this could be sensed in the experience of a woman without a man, or understood as ( \text{yin without yang} )</td>
</tr>
<tr>
<td>( e^{i\pi} - 0 = -1 )</td>
<td>Suggests that disassociation from the &quot;x-factor&quot;, of that which implies integrative globality, equates with a problematic condition of &quot;unity bereft&quot; -- a negative sense of unity</td>
</tr>
</tbody>
</table>

**Body knowledge: understanding without proof**

**Conceptual metaphors:** Comprehension of the subtle implications of the Euler equation above is the culminating focus of the arguments, from the perspective of cognitive psychology, of George Lakoff and Rafael Núñez (Where Mathematics Comes From: how the embodied mind brings mathematics into being, 2000). For them mathematics is based on "conceptual metaphors" that are a cognitive mechanism for allowing us to reason about one kind of thing as if it were another. As such it is an inference-preserving cross-domain mapping.

The authors argue that mathematics consists of metaphor piled on metaphor, blended and transformed, so people often do not realize the basis of it all. The Euler identity is then a concept that reflects a cognitive structure unique to humans (or less specifically to a narrow range of beings similar to humans). This argument is the focus of a comment by Robert J. Chassell (Understanding Without Proof. RattleSnake Enterprises, 2004). He usefully adds to the above remarks the sense that:

- **one**, is the amount of a single instance -- thus usefully to be associated with the typical one-off manner in which a strategic initiative is undertaken;
- **zero**, is that which is not there -- thus reinforcing the arguments above relating to the experience of being bereft and left with nothing, following the strategic initiative.

**** Deacon ???

**Paradox:** With regard to the comprehensibility of the Euler identity, Chassell quotes the mathematician Benjamin Peirce, at the end of a lecture in which he proved that identity:

> Gentlemen, that is surely true, it is absolutely paradoxical; we cannot understand it, and we don't know what it means. But we have proved it, and therefore we know it must be the truth. (Quotation from Edward Kasner and J. Newman, Mathematicas and the Imagination, New York 1940)

This suggests that the theme of paradox merits recognition in the quest for sustainable globalization and in relation to the nature of "nothingness" in that context.

**Requisite childlike insight?:** Chassell notes that, according to Lakoff and Núñez, comprehesion of such an identity is enabled by four "grounding" metaphors, namely metaphors based on experiences many have had as children. This recalls the separate discussion (Requisite Childlike Cognition for "Heavenly "Integration? 2012). The experiences are:

- adding and taking away objects from a collection (playing with pebbles);
- construction of a larger whole from smaller objects (playing with blocks);
- measuring the width or height of something (by stretching our hands to the ends of the object or standing up to see how high it is);
- moving from one place to another (by crawling or walking).

These experiences provide us with four metaphors that work with arithmetic: four inference-preserving cross-domain mapping mechanisms that work consistently with each other and the world. As such it might be fruitful to explore their relation to the archetypal morphologies identified by Rene Thom (Structural Stability and Morphogenesis: an outline of a general theory of models, 1972), as
Lakoff and Núñez argue that the Euler identity makes sense if, but only if, we understand that mathematics consists of the metaphorical extension of familiar notions into unfamiliar areas. In effect, despite its apparent challenge to comprehension, they assert an equivalence between the Euler identity and basic cognitive processes. This follows their earlier collaboration (Philosophy In The Flesh: the Embodied Mind and its Challenge to Western Thought, 1999).

Body thinking: The dependence of understanding on metaphors framed by body movement can be associated with the work of a colleague of Lakoff, namely Mark Johnson (The Meaning of the Body: aesthetics of human understanding, 2007). The focus is also a preoccupation of Maxine Sheets-Johnson (The Primacy of Movement, 1999; The Corporeal Turn: an interdisciplinary reader, 2009).

So-called "body thinking" has also been been reviewed as characteristic of ancient China by Kuang-Ming Wu (On Chinese Body Thinking: a cultural hermeneutic, 1996). He argues that Chinese thinking is concrete rather than formal and abstract, and this is gathered in a variety of ways under the symbol "body thinking". As now argued from a Western perspective, the root of the metaphor is that the human body has a kind of intelligence in its most basic functions.

The metaphor of body thinking is extended far beyond bodily functions in the ordinary sense to personal and communal life, to social functions and to cultivation of the arts of civilization. As the metaphor is extended, the way to stay concrete in thinking with subtlety becomes a kind of ironic play, a natural adeptness at saying things with silences. Play and indirection are the roads around formalism and abstraction. Western formal thinking, he argues, can be sharpened by Chinese body thinking to exhibit spontaneity and to produce healthy human thought in a community of cultural variety.

**Comprehension of Euler formula:** In the light of such insights, the question of how to respond to the cognitive challenge of "nothing" and the aspirations to more "global" insight might be answered by the exploration of metaphors with which people are widely familiar -- drawing on experience as children (as suggested in the separate discussion (Requisite Childlike Cognition for Integration of "Heaven"? 2012). As widely recognized, 99% of those who can catch a ball have no "feel" for the mathematics by which the process is described.

For one blogger (I challenge you to explain [the beauty of] Euler's Equation in terms I can understand):

> The first key idea to understand is that complex numbers are nothing more than rotations and scalings. If you know that two left turns make a U-turn, then you already understand that $i^2 = -1$... Once that's in place, it's not so hard to understand Euler's theorem. All it really says is that as you swing a stick in a circle, the direction in which the end of the stick is moving at any moment is 90 degrees rotated from the direction in which the stick is pointing. Is that beautiful? Well, it's certainly true. And if you put it in certain symbols, it looks mystical and lofty and so on. But it's a terribly simple theorem; so simple small children understand it, just without understanding that they understand it. [emphasis added]

The example given suggests comprehension of the classical coordination challenge of rubbing the stomach whilst rubbing or patting the head. The strategic initiatives in support of this merit consideration from a similar perspective.

Notably in the light of the name given to it, it is tempting to associate a degree of recognition of "imagination" with the "imaginary unit" $i$ - if only because imagination is required in reflecting on what that unit might possibly mean. Imagination is of course significant in the play of children and their elders. Surprisingly it is also significant in the call for "imagination" and "imaginative thinking!" in response to global crisis. It would also seem to be appropriate to consider the function of "intuition" in imaginative innovation. This argument ties in with the role variously accorded to "correspondences" by both the arts and the sciences, as discussed separately (Theories of Correspondences -- and potential equivalences between them in correlative thinking, 2007), notably in the light of their role in the mathematics of so-called "moonshine theory".

The notable absence of imaginative thinking was recognized by the US Senate (Learning from the 9/11 response: groupthink and failure of imagination, 2005). In the same mode, e offers the possibility of giving a degree of conceptual space to enigma -- otherwise conventionally excluded as necessarily meaningless and irrelevant. This would be consistent with the highlighting by Donald Rumsfeld of the "known unknowns" and the "unknown knowns", as separately discussed (Unknown Undoing: challenge of incomprehensibility of systemic neglect, 2008).

**Embodiment openness in a toroidal dynamic**

Recent research has endeavoured to reframe global challenges in terms of nine "planetary boundaries", as discussed separately in relation to analogous psychosocial boundaries (Recognizing the Psychosocial Boundaries of Remedial Action: constraints on ensuring a safe operating space for humanity, 2009). Of relevance to possible use of a toroidal metaphor is the association of a "doughnut" with those boundaries by Kate Raworth of Oxfam (Can we live inside the doughnut? Why the world needs planetary and social boundaries, 2012; A Safe and Just Space for Humanity: can we live within the doughnut?, 2012; Between social and planetary boundaries, Global Transition, 2012). This is intended to encourage public debate in anticipation of the forthcoming the UN Conference on Sustainable Development (Rio+20). Raworth argues that between the social foundation and the environmental ceiling lies a space - shaped like a doughnut - which is the safe and just space for humanity.

"Toroidal polyhedra": A clue to the dynamics associated with any "container" for belief is provided by mapping the comprehensive set of 64 "changes" traditionally identified in the Chinese Book of Changes (I Ching) onto a drilled truncated cube -- a so-called drilled toroid, as discussed separately (In-forming the Chalice as an Integrative Cognitive Dynamic: Sustaining the Holy Grail of Global Governance,
This polyhedral form, with a degree of spherical symmetry, is virtually unique in having 64 edges to hold and configure those changes. This then gives a "global" sense of how confidence and belief are "managed" coherently in relation to the continuing dynamics of decision-making. The form is valuable in that it offers an explicit sense of "out-of-the-box" and the manner in which the central cubic "box" is subsumed by a more complex space.

Fig. 11: Drilled truncated cube -- a polyhedron approximating a torus, with 64 edges
with which I Ching conditions of change have been arbitrarily associated
(images generated using Stella Polyhedron Navigator, variously rotated with selected faces coloured or not)
(video ***)

The following set of images show how the various spheres are related to the structure

Fig. 12: Drilled truncated cube with circumsphere, midsphere and insphere
(images generated using Stella Polyhedron Navigator, with selective omission of polyhedral faces)
(video ***)

Spiralling around "nothingness" and "pointlessness": the design implications of phi

Spiral ambiguity: The above argument highlights the strategic reliance on the "linear" and the challenge of configuring distinct strategies to approximate to governance requiring "global" thinking -- through engaging with the dynamics of cycles and circularity (Ungovernability of Sustainable Global Democracy? Towards engaging appropriately with time, 2011).

Through failure to think "otherwise", many of the global crises are however characterized by "vicious cycles" (Web resources on "breaking the cycle", 2002). Worse, many of those cycles are recognized and experienced to be "spinning out of control". In this respect it is notable how readily "hurricane", "vortex" and "black hole" have been used as metaphors to describe conditions such as the recent (and ongoing) global financial crisis. People have readily described themselves as being "sucked into a black hole" -- notably of despair.

Ironically however, the spiral is valued as emblematic of desirable forms of growth and development -- epitomized by the use of the marine nautilus, both as a symbol of educational development and of strategic appropriateness (New Zealand Curriculum Nautilus, Nautilus Institute for Security and Sustainable Development). The question is how to benefit from this dynamic, as discussed separately (Enabling Governance through the Dynamics of Nature: exemplified by cognitive implication of vortices and helicoidal flow, 2010).

Fig. 13: Spiral forms

<table>
<thead>
<tr>
<th>Nautilus shell (cutaway)</th>
<th>Logarithmic spiral</th>
</tr>
</thead>
<tbody>
<tr>
<td>(adapted from image in Wikipedia profile)</td>
<td>(adapted from image in Wikipedia profile, with dashed circles to indicate the 3D &quot;cognitive habitat&quot; of the developing body within)</td>
</tr>
</tbody>
</table>
Aesthetic function: In the above discussion of pi, reference was made to its "aesthetic" function in reconciling the linear with the circular. An aesthetic function is also central to the so-called golden ratio symbolized by phi (F). In both mathematics and the arts, this is recognized when the ratio of the sum of two quantities to the larger quantity is equal to the ratio of the larger quantity to the smaller one.

The aesthetic function then merits attention with respect to appropriate "design", as variously argued by Christopher Alexander (Harmony-Seeking Computations: a science of non-classical dynamics based on the progressive evolution of the larger whole, International Journal for Unconventional Computing (IJUC), 5, 2009), with respect to locating a "place to be" in a global context, as separately discussed (Harmony-Comprehension and Wholeness-Extending, 2010).

Fibonacci: In mathematics, the ratio of sequential elements of the Fibonacci sequence approaches the golden ratio asymptotically. The golden ratio has been noted in some polygons and some polyhedra. The Fibonacci series has been recognized in the spiral phyllotaxis of plants. A Fibonacci spiral, based on that series, closely approximates the golden spiral -- a logarithmic spiral whose growth factor is the golden ratio.

In the belief that this proportion is aesthetically pleasing, many artists, architects and designers have proportioned their works to approximate the golden ratio -- especially in the form of the golden rectangle, in which the ratio of the longer side to the shorter is the golden ratio. The golden ratio and related numbers are used in the financial markets. The ratio is used in trading algorithms, applications and strategies. Interpretation of the Fibonacci numbers in technical analysis anticipates changes in trends as prices tend to be near lines created by the Fibonacci studies. Some typical forms include: the Fibonacci fan, the Fibonacci arc, Fibonacci retracement and the Fibonacci time extension (Fibonacci Numbers/Lines, Investopedia).

Phi and Pi: Given the argument above for recognition of the role of an "aesthetic" modality in responding to the cognitive challenges of the times, it is appropriate to note that the relationship between pi and phi -- as the two most famous numbers in the history of mathematics. This has been explored by a number of authors, as recorded on the website Pi, Phi and Fibonacci Numbers (GoldenNumbers.net). For example:

- Ed Oberg and Jay A. Johnson (The Pi-Phi Product) have developed a unique expression for the pi-phi product as a function of the number 2 and an expression they term the The Biwabik Sum. This is a function of phi, the set of all odd numbers (of which all primes, excluding 2, are a subset), and the set of all Fibonacci numbers

Phi and pi are exactly related to each other by a several formulae, even though both are irrational numbers (as noted by Radoslav Jovanovic, Archimedes' constant PI and the Golden Section, 2005)

\[ \pi = 5 \times \arctan(\sqrt{5} - 2) \]
\[ \pi = 5 \times \arctan(\sqrt{5} - 2) \]

Designing patterns of "something" and "nothing" spiralling around an elusive "point": The construction of the Fibonacci spiral has been discussed separately as providing a framework to hold an evolving pattern of distinctions (Construction of Fibonacci spiral as providing an open-ended integrative framework, 2010). This lends itself to experimental attribution of the Chinese encoding of patterns of systemic insights, as shown in the following image (Fig. 14) derived separately (Experimental attribution of binary codes to Fibonacci pattern, 2010). This is as an exploration of the relevance of the pattern to sustainable governance (Fibonacci's magic carpet of games to be played for sustainable global governance, 2010).
Fig. 15: Euler's spiral or Clothoid
(See also screen shots from an interactive representation
The spiral converges to the centre of the holes in the image
as the length of the spiral (measured from the origin) tends to positive or negative infinity.

As previously suggested, following this pattern, Fig. 14 may itself be inverted as shown in Fig. 16.

Inverting the spiral: The spiral form -- itself centred on an elusive point -- raises the question of its further evolution, namely of "where it is going". This is more apparent in three dimensions, as in the case of the shell of the marine Nautilus which dwells "along" the spiral, within a tube of circular cross-section. Its growth is "out" of the circular opening at the end of the spiral -- to the extent that it is able to grow further. It is in this sense that spiral horns have had valued symbolic associations -- notably with unlimited abundance, as the "horn of plenty: (or cornucopia)***trumpet The Hindu shankha conch horn is still used as a trumpet in Hindu ritual, and was used as a war trumpet in the past.

Framed otherwise in the light of weather systems, characterized by the spiralling cyclones and anti-cyclones on the surface of the planetary sphere, the question is the nature of the larger context in which such flow processes are interrelated. Depending on the perspective of the observer, spiral galaxies spin both clockwise and anti-clockwise within the universe -- as do black holes.

The spiral in Fig. 14 may therefore be extended by inverting it, as indicated by the following image, reproduced from the context of a related argument (Designing Global Self-governance for the Future: patterns of dynamic integration of the netherworld, 2010). As noted there, Euler spirals (also known as spiros, clothoids or Cornu spirals) are curves whose curvature changes linearly with the curve length (the curvature of a circular curve is equal to the reciprocal of the radius). Of relevance to the above argument, they are also widely used in transition curve design in railroad and highway engineering for connecting and transiting the geometry between a tangent and a circular curve.
Alternation -- That which is, is not. That which is not, is
-- perhaps indicative of the holomovement described by David Bohm

Encompassing: "squaring the circle" through the "circling the square"? The challenge of "squaring the circle" was formulated by by ancient geometers, namely that of constructing a square with the same area as a given circle by using only a finite number of steps with compass and straightedge.

From a a Jungian psychological viewpoint, one of the perennial problems for humans is, that of squaring the circle. By this is to be understood that the human self-development task, represented by the circle, is to have optimally developed four functions (thinking, feeling, sensing, and intuition). Self-development begins by developing this square. The traditional Masonic emblem -- the square and compasses -- is taken as a symbolic representation of the solution to the Jungian problem: a freemason is then one who learns to square the circle (William Steve Barkle, Musings on the Geometric Properties of the Square and Compasses, Review of Freemasonry; Paul Calter, Squaring the Circle: geometry in art and architecture, 2008).

Fig. 17: Spiralling "out-of-the-box" -- or into it?

Recognizing the "point" and reframing "nothingness": discontinuity and surprise

This argument has explored the widespread metaphorical use of "point" and "line" in relation to the engagement with "nothing". The associated geometry has been presented as a "vehicle" or "channel for thought and cognitive processes -- whether for "point" or "line". It might then be asked whether there is a degree of transition in the way experience is "carried" between:

<table>
<thead>
<tr>
<th>Cognitive implications of point and line as metaphors</th>
<th>Line</th>
<th>Point</th>
</tr>
</thead>
<tbody>
<tr>
<td>perceiving a connection</td>
<td>recognizing a point</td>
<td></td>
</tr>
<tr>
<td>pursuing a &quot;line of thought&quot;</td>
<td>making a point (or a pointed remark)</td>
<td></td>
</tr>
<tr>
<td>articulating a &quot;line of argument&quot;</td>
<td>formulating text of a point (for example, a &quot;tweet&quot;)</td>
<td></td>
</tr>
<tr>
<td>formulating text of an argument (for example, succession of &quot;tweets&quot;)</td>
<td>textual punctuation mark (ensuring closure)</td>
<td></td>
</tr>
<tr>
<td>drawing a line (as in a concept map)</td>
<td>presentation of an array of &quot;bullet points&quot; (Power Point)</td>
<td></td>
</tr>
<tr>
<td>social alignment (as in &quot;taking a line&quot;, &quot;pursuing a line&quot;)</td>
<td>leading as &quot;point person&quot;</td>
<td></td>
</tr>
<tr>
<td>being upstanding (as in relation to an issue)</td>
<td>exemplifying the point</td>
<td></td>
</tr>
<tr>
<td>being an exemplar (with respect to response to an issue)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The cognitive implications of thought can indeed be "embodied" by an individual (or a group), as the culmination of these transitions. However the approach suggests that, in some of the earlier phases of the transition, the "logic" can be embodied in the geometry and its development, as is characteristic of certain arts exploring patterns, most notably design.

Questioning the "point": It is in this sense that the actual "geometry" of the question mark may carry further insight, notably by pairing it with the inverted question mark used to open interrogation in some languages (for example, ¿Qué hora es? meaning What time is it?). Of related interest are other variants: the percontation point (or rhetorical question mark), the irony mark, and the mirrored question mark. These are all indicative of qualities of the cognitive uncertainty, surprise and discontinuity in the despairing experience of "nothing" -- and its anticipation. The experience may be further articulated by pairing with an exclamation mark -- as in the the interrobang.

The pairing presented in Fig. 18 is considered here because of the manner in which it can be presented such as to echo the form of the double spiral presented above -- with the significance that it encompasses. However the pairing would also seem to be appropriate to the cognitive challenge of "nothingness" -- given the existential questions it raises.

Such pairing has been used experimentally in a related argument (¿ Defining the objective ≠ Refining the subjective ?! Explaining reality ≠ Embodying realization, 2011; ¿ Higher Education ≠ Meta-education ?! 2011). This is consistent with the existential ambiguities
Summit on Sustainable Development worthwhile

Aspects of this argument were explored in relation to the "waking up", will humanity know whether it was a civilization dreaming it was explored (The higher dimensionality of the connectivity of any "cognitive wormhole" linking the contrasting "points" of view is consistent with a call" for humanity, and references to "living in a dream". The challenge is the comprehension of the "Transformation of Things".

The currently carefree attitude of global society might be fruitfully compared to Zhuangzi -- despite various articulations of a "wake-up conundrum of Togetherness: a cultural hermeneutic raised by the above argument (Beyond rational thinking: Fig. 18 is delightfully reminiscent of the pattern of the classical symbolic representation of Tao (Fig. 19) and the cognitive challenges it implies. Comprehension of it is notably enabled by the curious "questions" characterized by the Zen koan -- the paradoxical meaning of which cannot be appropriately understood by rational thinking but may be accessible through intuition or lateral thinking. One classical collection of such koans is the so-called Gateless Gate. Such a framing would seem to be appropriate to the nature of the global cognitive crisis, and the associated sense of "pointlessness" in the face of "nothing".

The wormhole, as the connection between "point" and "counterpoint" of the paired questions, might be usefully understood as a form geometrically analogous to an Euler spiral (Fig. 15). This suggests a way of thinking of a wormhole through "semantic hyperspace", as might be explored in terms of the work of Rene Thom discussed separately (Beyond rational thinking: see Thom, 2002). That exercise is now to be repeated in Rio de Janiero (2012), as noted above.

Especially intriguing is the possibility that the cognitive "wormhole" suggested by Fig. 18b and Fig. 18c relates to the classic mirror conundrum of Chuang Tzu (Zhuangzi), as discussed by Kuang-Ming Wu (The Butterfly as Companion: meditations on the first three chapters of the Chuang Tzu, 1990):

"Zhuangzi dreamed he was a butterfly"

Once Zhuangzi dreamt he was a butterfly, a butterfly flitting and fluttering around, happy with himself and doing as he pleased. He didn't know he was Zhuangzi.

Suddenly he woke up and there he was, solid and unmistakable Zhuangzi. But he didn't know if he was Zhuangzi who had dreamt he was a butterfly, or a butterfly dreaming he was Zhuangzi.

Between Zhuangzi and a butterfly there must be some distinction! This is called the Transformation of Things.

The currently carefree attitude of global society might be fruitfully compared to Zhuangzi -- despite various articulations of a "wake-up call" for humanity, and references to "living in a dream". The challenge is the comprehension of the "Transformation of Things".

The higher dimensionality of the connectivity of any "cognitive wormhole" linking the contrasting "points" of view is consistent with what is recognized in physics as quantum entanglement. The paradoxical cognitive possibilities of entangled mirroring can be variously explored (Hyperspace Clues to the Psychology of the Pattern that Connects in the light of 81 Tao Te Ching insights, 2003). After "waking up", will humanity know whether it was a civilization dreaming it was a "butterfly", or a "butterfly" dreaming it was humanity. Aspects of this argument were explored in relation to the UN Earth Summit 2002 (My Reflecting Mirror World: making my World Summit on Sustainable Development worthwhile, 2002). That exercise is now to be repeated in Rio de Janiero (2012), as noted above.

<table>
<thead>
<tr>
<th>Fig. 18: Design experiments in pairing question marks</th>
<th>Fig. 18a</th>
<th>Fig. 18b</th>
<th>Fig. 18c</th>
</tr>
</thead>
<tbody>
<tr>
<td>to highlight cognitive implications of mirroring and potential &quot;entanglement&quot; of &quot;points&quot;</td>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
<td><img src="image3.png" alt="Image" /></td>
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</tbody>
</table>

**Discontinuity**: Of particular interest is the implication of the discontinuity in the design of the question mark itself -- between the semi-circular portion and the "point", possibly with an intermediary linear portion in some typographical representations. In its singular form, the circle is incomplete and highlights the dilemma of how to complete it -- or the point of doing so.

Pairing with the inverted mark highlights the relevance of a form of cognitive "netherworld" -- the "dark side" of the question -- calling for consideration to achieve a degree of completion, as discussed separately (Designing Global Self-governance for the Future: patterns of dynamic integration of the netherworld, 2010). This highlights the existence of a complementary "point" -- perhaps a counterpoint, fruitfully explored in terms of musical harmony. The two "points" may then be understood as implying a "hidden" cognitive relationship as suggested by Fig. 18b.

Metaphorically this could be understood as a form of cognitive "wormhole" of different dimensionality. Its nature is suggested by the cognitive experience of humour, surrealism and surprise (as discussed below). The role of humour in enabling connectivity, even under highly problematic conditions, is widely recognized (Humour and Play-Fullness: essential integrative processes in governance, religion and transdisciplinarity, 2005).

The wormhole, as the connection between "point" and "counterpoint" of the paired questions, might be usefully understood as a form geometrically analogous to an Euler spiral (Fig. 15). This suggests a way of thinking of a wormhole through "semantic hyperspace", as might be explored in terms of the work of Rene Thom discussed separately (Beyond rational thinking: see Thom, 2002). That exercise is now to be repeated in Rio de Janiero (2012), as noted above.

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Engagement with the "Other": The explicit form of the paired question marks of Fig. 18, and of the symbol of the Tao, obscures an implicit reality which they also embody to a degree. This is associated with the nature and degree of cognitive denial of otherness which is only too evident as fundamental to global dynamics (Reframing the Dynamics of Engaging with Otherness, 2011).

Ironically the nature of this denial is readily explained and mapped by the Tao symbol through one of the most widely recognized experiences, namely snoring (Snoring of The Other: a politically relevant psycho-spiritual metaphor? 2006). This is considered with respect to the following: Tao of Snoring?: Higher dimensionality necessary for sustainable sharing?; Mirroring and mixing metaphors; Mirror symmetry within the semiosphere.

The symbol fruitfully holds the sense in which one modality communicates by a form of inversion with another. This is evident with respect to the switch in colour between black and white in the Tao symbol -- present to a degree in the manner in which the paired question marks are presented. The nature of the "point" is transformed within that dynamic.

Enabling the transition between linear and circular: As reinforced by the above images, there is a case for exploring further the possibilities of smooth transition between "linearity" and "circularity" in the framing of social processes and relationships. This possibility is developed further in a separate document (Orbiting Round Nothingness across Communication Space: possibility of an "Inter-other Transition Network", 2012) in the light of the clothoid form introduced above.

Dynamic reframing of surprise: Black Swan / White Raven

Swans: The nature of surprise of relevance to the supposed cognitive rationality of global governance has been made evident by black swan theory, notably as articulated by Nassim Nicholas Taleb (The Black Swan: The Impact of the Highly Improbable, 2007). A black swan is extremely rare in most parts of the world.

Ravens: Of potential relevance to wider understanding is the myth common to many cultures of the relation between the "white raven" and the "black raven" -- with a white raven being extremely rare. The story of the white raven is widely recognized among raven mythologies in many indigenous cultures. The tale in all such contexts starts with the raven being white in colour. By the end of the story it has turned black, whether as a punishment offensive to the gods or as an act of self sacrifice. Whether the story has raven earning his black feathers by being a saviour or otherwise, all of the mythologies say the true and highest form of raven is white. In one variant:

"... In our oral traditions, Raven was originally white. There’s days and days of stories of Raven as he … haphazardly brings the world into existence as we know it. Through that process he steals the sun and the moon from their caretakers, flies through the smoke hole in the longhouse and brings light to the world. And when he flew through the smoke hole, he became black. Some people view the white raven as being Raven in his truest form, having to undergone challenges and sacrifices to bring about better good for people."

Swan and Raven: Curiously, but appropriately, the "black swan" and the "white raven" have been paired in a modern anthology (Ellen Datlow and Terri Windling, Black Swan, White Raven, 2008). This is a creative re-telling of fairy tales by modern authors. However, rather being children's stories, many depict sex, violence, and other subjects - just like the original fairy tales, before they were whitewashed:

Fairy tales have an odd sort of fundamental appeal. They're stories of love and loss, revenge and justice, royalty and peasantry, mundanity and magic. Some have a moral; others are told to explain natural events. Many started out as popular folk tales. Most address what happens when ordinary people meet up with the world of the extraordinary.

Eagles: The use of "birds" (rather than geometry) as carriers of the founding myths of a culture is relevant with respect to Western culture at this time, given its origins in Ancient Greece. According to that mythology, Zeus sent out two eagles to fly across the world to meet at its center, the "navel" of the world. Hemispherical omphalos stones were used to denote this "point". These stones were once considered the holiest object at various oracle centers -- in all the lands bordering the Mediterranean Sea, the most well known being at Delphi in the centre of the Temple of Apollo (the god of light, music, geometry, and harmony, whose nature most closely reflects the highest realizations of the Greek spirit).

The main characteristic of the omphalos was the ability to allow direct communication with the gods. The stone is often shown covered by an unusual net-like pattern [images] -- perhaps the "flight-path" of the "eagles". This pattern has been considered in relation to the spherical geometry of tensegrity structures of relevance to global governance (Warp and Weft: Governance through Alternation, 2002).

"Point-making" by "birds": Birds then offer a more elegant way of considering both "point" and "pointlessness" through the manner in which they fly into a space where they are recognized and to whose framing in the moment they contribute -- effectively "punctuating" it with their song. They may perch awhile -- perhaps in a "line" -- before flying "away". Dialogue can be framed in this way -- especially in cyberspace, as discussed separately (Flocking behaviour and the dynamics of gated conceptual communities, 2004). The metaphor can
be extended -- questionably -- to consider the role of any replicas made to commemorate their passage and that of their "fewmets".

**Myth mining**: Whether ravens, swans or eagles -- or butterflies -- the cultural understanding of the dynamics held by such mythical tales could be fruitfully considered as variously "encoded" by interpretations of the paired question marks or the Tao symbol (above). As previously argued, there is a case for mining and correlating civilizational metaphors (*Enhancing the Quality of Knowing through Integration of East-West metaphors*, 2000). This is especially the case in the light of worldwide popular attention devoted to the blockbuster movies of modern myth-making (*Relevance of Mythopoeic Insights to Global Challenges: cognitive integration implied by the Lord of the Rings*, 2009). The metaphor search engine *Yossarian Lives!* should prove relevant to such an undertaking.

Despite the fanciful nature of such myths, it is ironic that governance is typically constrained by political processes which value one "wing" -- right or left -- over the other. The preoccupations and style of each "wing" tend to be systematically deprecated by the other -- suggesting an aspiration to an improbable form of "one-wing" governance, as variously explored (Madjuranga Rathnayake, *Flying with One Wing and the Countryless*, Sri Lanka Guardian, May 2010; Stanley D. Williams, *Trying to Fly with One Wing*, Catholic Lane, 13 June 2011). As with the painful *phantom limb* sensations, following loss of a limb, it may be supposed that the body politic could experience painful "phantom wing" sensations under such circumstances -- especially in any aspirations of the international community to sustainable global governance. Ironically "phantom wing" sensations have been attributed by individuals to the absence of (angelic) wings.

NB: See separate presentation of relevant bibliographical references.

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