Polyhedral Empowerment of Networks through Symmetry

Psycho-social implications for organization and global governance


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Context

In the further exploration of the possibilities highlighted in the above-mentioned papers, reference is made here to the current applications of "polyhedral approaches" to networks, their operation, and to their significance for information organization in situations calling for higher orders of efficiency and robustness -- and to the insights offered for new approaches to psycho-social organization.

The main emphasis below has been to indicate fields of study and application which may not necessarily be well-connected, however relevant they may be with respect to any potential psycho-social implications. Given that the concept of "network" has proven over past decades to be as significant as a metaphor for social organization as it has as an analytical framework for the development of such organization, it is possible that the potential of "polyhedra" should be similarly understood.

The basic argument is that in psycho-social usage "network" is relatively unstructured and has not achieved much of what was hoped initially in contrasting it with hierarchicical modes of organization. Whilst seemingly quite unrelated, the faces and edges linking vertices of any 3-dimensional polyhedron can be mapped in 2-dimensions as a network (a polyhedral net) -- by "unwrapping" the polyhedron.

Studies of social networks show that desirable properties such as robustness and information transfer efficiencies can be achieved with networks that take the form of polyhedra having properties such as symmetry. Symmetrical polyhedra in 3-dimensions are those of greatest aesthetic appeal and are intuitively comprehensible as a whole, despite their possible complexity.

There is therefore a case for considering how, in the light of the various polyhedral approaches (considered below), networks might be "polyhedrally empowered" by using polyhedra as structural (or dynamic) templates. Of particular related interest is the degree to which
complex multicriteria decision-making is now dependent on such approaches -- suggesting that the comprehensibility and communicability of a solution to any strategic dilemma might be associated with a polyhedral form reflecting its "goodness of fit" as a pattern in a design sense. This would then have important implications for governance.

**Challenge of network connectivity and networking efficiencies**

"**Tensing networks**": In response to early optimism regarding the merits of social "networking", in contrast to the problematic aspects of hierarchical social organization, attention was focused on the inefficiencies of untensed networks and associated "networking diseases" (Tensing Associative Networks to contain the Fragmentation and Erosion of Collective Memory, 1980; Implementing Principles by Balancing Configurations of Functions: a tensegrity organization approach, 1979; Tensed Networks: balancing and focusing network dynamics in response to networking diseases, 1978). These concerns resulted in a continuing preoccupation with tensegrity organization, namely ensuring a degree of tensional integrity within networks (From Networking to Tensegrity Organization, 1984; Documents relating to Networking, Tensegrity, Virtual Organization).

Recent developments with respect to tensegrity as an extension of the focus on polyhedra, notably relevant software, are discussed elsewhere (Psycho-social operationalization of polyhedra through tensegry representation, 2008).

**Network graphs as polyhedra**: Just as three-dimensional polyhedra may be represented in two dimensions as a network -- a polyhedral net -- so there have been explorations of the value of representing networks by polyhedra (Branko Grünbaum, Graphs of Polyhedra: Polyhedra as Graphs, Discrete Mathematics, 2007). Of particular interest has been the generic character of network connectivity (M. Grötschel, C.L. Monma and M. Stoer, A Polyhedral Approach to Network Connectivity Problems, 1992).

**Optimal networks -- classification of polyhedra**: As part of the quest for more useful networks, efforts have been made to classify polyhedra.

Schläfli symbol: Michael J. Bucknum and Eduardo A. Castro (Geometrical-Topological Correlation in Structures, Nature Precedings, March 2008) describe the topological indexes of polygonality, n, and connectivity, p, which can be identified for various structures, including the 2-dimensional (2D) tessellations and the 3-dimensional (3D) crystalline patterns. The ordered pair (n, p), called the Schläfli symbol, that is descriptive of the topology of each and every distinct structure, is identified and used to illustrate a Schläfli-space, entries of which have the coordinates n and p, that can be employed to map the innumerable structures, and to identify relations between and among these structures graphically, so that absolute identities and locations can be ascribed to them.

Wells fundamental polyhedra metric: Bucknum and Castro use the work of A.F. Wells (Three Dimensional Nets and Polyhedra, 1977) -- his polyhedral metric, structural correspondence principle and morphological principle to derive the polyhedral and 2D and 3D metrics. They note that Wells, as "a master of applied topology" derived more than 100 novel networks.

Wythoff construction: In geometry, a Wythoff construction is a method for constructing a uniform polyhedron or plane tiling. It is often referred to as Wythoff's kaleidoscopic construction. It is based on the idea of tiling a sphere, with spherical triangles. If three mirrors were to be arranged so that their planes intersected at a single point, then the mirrors would enclose a spherical triangle on the surface of any sphere centered on that point and repeated reflections would produce a multitude of copies of the triangle. If the angles of the spherical triangle are chosen appropriately, the triangles will tile the sphere, one or more times. A Wythoff symbol is a short-hand notation for naming the regular and semiregular polyhedra using a kaleidoscopic construction, by representing them as tilings on the surface of a sphere, Euclidean plane, or hyperbolic plane (see examples).

Greg Egan (Wythoff, 2002) provides an applet that displays uniform polyhedra, using Wythoff's kaleidoscopic construction to compute the locations of the vertices. By clicking on the applet, 74 of the 80 possible uniform polyhedra (including single examples from each of the five infinite classes of prisms and antiprisms) are displayed.

Such work focuses the question of whether the range of polyhedra, especially with some degree of symmetry, constitute a rich repertoire for psycho-social organization appropriate to various conditions, as previously argued (Polyhedral Pattern Language: software facilitation of emergence, representation and transformation of psycho-social organization, 2008).

**Optimal networks -- information traffic**: There has naturally been a major interest in optimizing the flow of information through networks, notably in telecommunications. Raul J. Mondragon (Optimal Networks, Congestion and Braess' Paradox, 2007), for example, explores how to deliver information efficiently in a communications network and how to build networks to perform this function -- notably by re-wiring them. Following Dekker and Colbert's work, they seek a compromise between robust networks and optimal networks:

- node connectivity = minimum number of nodes needed to remove to obtain a disconnected network
- link connectivity = minimum number of links needed to remove to obtain a disconnected network.

which leads to a focus on regular and symmetric graphs in which the nodes are all similarly linked. They cite the work of L. Donetti et al. (Optimal network topologies: Expanders, Cages, Ramamujan graphs, Entangled networks and all that, 2006; Entangled Networks, Synchronization, and Optimal Network Topology, 2005) in discussing the network congestion under load in various configurations.

Such considerations are especially relevant to consideration of information (if not "knowledge" or "wisdom") flows in psycho-social networks. Given the increasing degree of enablement offered by the internet and the web, to what extent could the challenges of information overload and information underuse be circumvented by a polyhedral approach to optimization of such networks? In principle this is highly relevant to the challenges of a learning society and the threats to collective memory (Societal Learning and the Erosion of...
Collective Memory a critique of the Club of Rome Report: No Limits to Learning, 1980).

Optimal networks -- commodity distribution: Gábor Rétvári, József J. Bró and Tibor Cinkler (Fairness in Capacitated Networks: a Polyhedral Approach) address the problem of allocating scarce resources in a network so that every user gets a fair share, for some reasonable definition of fairness. For example, a fair allocation would be such that every user gets the same share, and the allocation is maximal in the sense that there does not exist any larger, even and feasible allocation. We shall focus on the fair allocation problem that arises most often in networking: compute a fair rate at which users can send data in a telecommunications network, whose links are of limited capacity. The authors show that We show that the set of throughput configurations realizable in a capacitated network makes up a polyhedron, which gives rise to a max-min fair allocation completely analogous to the conventional one. An algorithm to compute this polyhedron is also presented, whose viability is demonstrated by comprehensive evaluation studies.

Gabriella Muratore (Polyhedral approaches to survivable network design, 1999) studies the problem of designing a cost-efficient multicommodity flow network with survivability features and the geometrical structure of several polyhedra arising in this context. For some of these polyhedra it proved possible to give a complete description by extreme points and by facets, while for others the complete description was given by extreme points. Several classes of facet-defining inequalities were identified.

It might be inferred that a more sophisticated approach to "fairness" is what is required in response to the variety of forms of "unfairness" that drive social unrest and cycles of violence -- whether collectively or within the interpersonal networks.

Polyhedral approaches to social network analysis

The interest in the network structure of organizations, especially those of a potentially criminal or terrorist nature, has increased manifold as a result of the information revolution, as noted by B. Balasundaram, et al (Clique Relaxations in Social Network Analysis: The Maximum k-plex Problem, 2006). Their paper introduces and studies the maximum k-plex problem. This arises in analysis of cohesion in social networks and is often used to explain and develop sociological theories. Members of a cohesive subgroup tend to share information, have homogeneity of thought, identity, beliefs, behavior, even food habits and illnesses. It is also believed to influence emergence of consensus among group members. The approach is relevant to the study of degrees of connectivity amongst sets of websites. It may be used in organizational management to study organizational structure to suggest better work practices and improve communication and work flow.

Their study helpfully summarizes the distinction in the literature between three important structural properties expected of a cohesive subgroup that are idealized by models of clique models idealize:

- familiarity (each vertex has many neighbors and only a few strangers in the group),
- reachability (a low diameter, facilitating fast communication between the group members) and
- robustness (high connectivity, making it difficult to destroy the group by removing members).

Different models may relax relax different aspects of a cohesive subgroup:

- a distance based model called k-clique
- a diameter based model called k-club
- a variant called k-clan
- a degree based model called k-pex.

This model relaxes familiarity within a cohesive subgroup and implicitly provides reachability and robustness. As the authors note:

In spite of its potential applicability to a number of important practical situations, the optimization problems concerned with finding large k-plexes in a graph have not been studied from the mathematical programming perspective. It is surprising that since the introduction of the k-plex model and establishing its basic mathematical properties in the late 70's, it has been completely overlooked in mathematics, mathematical programming and computer science literature.

It is curious that there are few memorable instances where such sophisticated tools have enabled better networks to form. The current explosion of "social networking" over the web, for example, does not appear to have benefitted from such insights. The relevant Wikipedia entry exemplifies the point with the statement: "Not to be confused with social network analysis, a type of social scientific model". It is however clear that the insights have been considered highly relevant in tracking criminal networks and terrorist suspects. Whereas they enable detection of problematic nodes against which security measures can be taken, they have not has yet enabled community-building in practice.

Polyhedral networks: designing for robustness and survivability

Curiously it would appear that the array of network analysis skills has been most significantly applied "defensively" in response to possible vulnerabilities of vital networks -- whether to protect against them or to exploit them. For example, the study by Jonathan T. Hamill (Analysis of Layered Social Networks, Air Force Institute of Technology, 2006) is concerned with prevention of near-term terrorist attacks:

To aid in this understanding, operations research, sociological, and behavioral theory relevant to the study of social networks are applied, thereby providing theoretical foundations for new and useful methodologies to analyze non-cooperative organizations. Such organizations are defined as those trying to hide their structures or are unwilling to provide information regarding their operations; examples include criminal networks, secret societies, and, most importantly, clandestine terrorist organizations.
As noted by Anthony H. Dekker and Bernard D. Colbert (Network Robustness and Graph Topology, 2004; Network Robustness for Critical Infrastructure Networks, 2008):

Two important recent trends in military and civilian communications have been the increasing tendency to base operations around an internal network, and the increasing threats to communications infrastructure. This combination of factors makes it important to study the robustness of network topologies. We use graph-theoretic concepts of connectivity to do this, and argue that node connectivity is the most useful such measure. We examine the relationship between node connectivity and network symmetry, and describe two conditions which robust networks should satisfy. To assist with the process of designing robust networks, we have developed a powerful network design and analysis tool called CAVALIER, which we briefly describe.

After reviewing a range of polyhedra, the authors conclude:

We have discussed the graph-theoretic concepts of node connectivity and link connectivity as measures of network robustness, and argued that node connectivity is most appropriate for modelling the robustness of network topologies in the face of possible node destruction. This is important both for military networks and for civilian networks facing possible terrorist activity.... We therefore suggest that military networks, or civilian communications backbones, be node-similar and optimally connected, with degree as high as feasible, diameter as low as feasible, symmetric if possible, and containing no large subrings.

The problem of robustness is vital to telecommunications networks as explored by Bernard Fortz (Design of Survivable Networks with Bounded Rings, 2000) using polyhedral analysis. The results obtained demonstrate how to use polyhedral theory for practical network design problems.

From a military perspective, a recurring theme in the literature is the "design of survivable communication networks" (M. Grötschel, C.L. Monma, M. Stoe, Polyhedral and Computational Investigations for Designing Communication Networks with High Survivability Requirements, 1992). This perspective is notably relevant to telecommunications networks (Arie Koster, Polyhedral Combinatorics to Solve Network Design Problems, Paper for 9th INFORMS Telecommunications Conference, 2008).

In the increasing concerns with sustainability, and the longer-term viability of catalytic social projects, the issue of what makes for robustness would appear to be vital -- faced with the tendency of projects to collapse once seed funding ceases. Such robustness is clearly also of importance faced with the prospect of partial or complete social collapse, if only in the event of disasters. Recent disasters have indicated the vulnerability of food and utility supply networks, for example.

Epistemic networks, simplicial complexes and polyhedra

In a society increasingly recognized to be significantly knowledge-based, social networks, concept networks and the manner of their apprehension are necessarily intimately intertwined -- beyond the challenge of the description of networks in the abstract and the flows of information through them. This interweaving has been remarkably explored by Camille Roth (Co-evolution in Epistemic Networks: reconstructing social complex systems, 2005). The author frames his discussion as follows:

Agents producing, manipulating, exchanging knowledge are forming as a whole a socio-semantic complex system: a complex system made of agents who work on and are influenced by semantic content, by flows of information in which they are fully immersed but, at the same time, on which they can have an impact and leave their footprints. Social psychologists and epistemologists, inter alia, have already a long history in studying the properties of such knowledge communities. Yet, the massive availability of informational content and the potential for extensive interactivity has made the focus slip from single 'groups of knowledge' to the entire 'society of knowledge'....

Here an 'epistemic community' is understood as a descriptive instance only, not as a coalition of people who have some interest to stay in the community: it is a set of agents who simply share the same knowledge concerns. Epistemologists traditionally describe a whole field of knowledge by characterizing and ordering its various epistemic communities, and they basically achieve this task by gathering communities in a hypergraph, which we call epistemic hypergraph. A hypergraph is a graph where edges can connect groups containing more than two nodes. We thus support the following thesis: the structure of a knowledge community, and in particular its epistemic hypergraph, is primarily produced by the co-evolution of agents and concepts.

In contrast to the graph theory tools typically deployed for social network analysis, Roth bases her analyses on the use of Galois lattices that may be termed 'concept lattices' in other contexts (Wille, 1992; Stumme, 2002). With regard to the above objective, Roth notes:

Given the assumptions, an adequate and efficient method for achieving this task consists in using Galois lattices. By checking the adequation between the resulting hypergraph and an empirical highlevel epistemological description of the knowledge community -- i.e. of the kind epistemologists would produce and work on -- we will confirm the validity of the projection.... This provides subsequently a formal way of partially defining the field of 'scientometrics', which consists in describing scientific field and paradigm evolution from low-level quantitative data.... More precisely, we will introduce a co-evolutionary framework based on a social network, a semantic network and a socio-semantic network; as such an epistemic network made of agents, concepts, and relationships between all of them.

With respect to polyhedra, Roth notes that the principles underlying use of Galois lattices (GLs) strongly relate to Q-analysis, notably that
of Ron Atkin on simplicial complexes:

Again, given a relation $R$ between two sets, Q-analysis introduces polyhedra such that for each object $s$ of the first set, the associated 'polyhedron' is made of vertices $c$ such that $s Rc$. The notion of 'maximal hub / maximal star' replaces that of closed couple (Johnson, 1986). However, while Galois lattices focus on the hierarchy between closed couples, Q-analysis is more interested in connected paths between polyhedra, by making an extensive use of equivalence classes of Q-connected components. In particular, two polyhedra sharing at least $Q+1$ vertices are Q-near, and polyhedra between which there is a chain of Q-near polyhedra are said to be Q-connected.

With respect to the structure of any knowledge community, Roth introduces a formal framework based on Galois lattices that categorizes epistemic communities automatically and hierarchically, rebuilding a whole community taxonomy in the form of a hypergraph of significant sub-communities. She argues that modeling social complex systems tends to require the introduction of co-evolutionary frameworks.

More generally, investigating the methodology of complex system science, we suggested that some high-level phenomena cannot be explained without a fundamental viewpoint change in not only low-level dynamics but also in the design of low-level objects themselves.

Roth notes that beyond the profusion of community-finding methods, often leaning towards AI-oriented clustering, an interesting issue concerns the representation of communities in an ordered fashion. She cites as examples of different techniques for producing and representing categorical structures: hierarchical clustering, Q-analysis, formal concept analysis, information theory, blockmodeling graph theory-based techniques, neural networks, association mining, and dynamic exploration of taxonomies.

In later work relevant to the co-evolution of social and knowledge networks, Camille Roth (Patterns and Processes in Socio-semantic Networks, 2007) notes:

Often, though, knowledge networks are treated like any other real network, with agents behaving in a way sometimes not much more complex than molecules. Even when the behavioral complexity of agents is taken into account, social network models seem to neglect epistemic features. Our goal is to emphasize the intertwining of social and semantic networks, both theoretically and empirically: we first suggest that binding these networks yields new kinds of patterns, showing notably how community structure may subsequently be appraised.

Also of note is R. Cowan, et al. (The Joint Dynamics of Networks and Knowledge, 2002), as well as the literature on the diffusion of innovation through networks.

It is however intriguing that Roth's central focus on "reconstructing social complex systems" might be understood purely as one of constructing a more adequate model of such a system rather than responding to the challenge of designing and facilitating the operation of more appropriate systems. This has long been a weakness of social network "analysis" which (as noted above) has not yet proven to be significant in designing better networks, whatever that might mean.

**Polyhedral dynamics and Q-analysis**

Polyhedral dynamics is a tool for representing network structure and behaviour. In introducing this field and its relevance to IIASA (International Institute for Applied Systems Analysis), J. Casti (Polyhedral Dynamics: the relevance of algebraic topology to human affairs, 1975) notes:

The most serious single methodological obstacle in the analysis of large-scale systems has been the lack of a suitable mathematical apparatus capable of describing the global features of a system, given information about local (subsystem) behavior. It is perhaps not surprising that the heavy emphasis placed upon the use of tools of analysis has yielded very meager fruits in this regard, since the methods of classical analysis are inherently local, being based upon such concepts as derivatives, infinitesimals, power series expansions, and so forth which are all concerned with behavior in the neighborhood of a point. What is surprising, however, is that, with few exceptions, the other main roots of mathematics - algebra and geometry - have not been tapped to provide a new set of tools for the system theorist to probe the murky depths of large, complex systems.

Casti notes that the approach was introduced by Ron Atkin (Mathematical Structure in Human Affairs, 1974) using ideas of algebraic topology.

By a very ingenious coupling of classical ideas in combinational topology and new notions of connectivity, patterns, and obstructions, this work presents a mathematical framework within which an extremely broad class of global systems questions can be precisely analyzed.

Elsewhere (Topological Methods for Social and Behavioural Systems, 1982) Casti notes:

Catastrophe theory focuses upon the structure present in smooth functions of several variables and provides a geometric
language for characterizing this structure. The language termed "q-analysis" ..., or "polyhedral dynamics" ..., offers a similar approach to the study of binary relations between finite sets of data. Thus, while catastrophe theory with its emphasis upon smooth functions, is heavily-flavored by the analytic tools of differential topology, q-analysis relies upon the ideas and methods of algebraic topology.

Atkin introduced polyhedral dynamics in terms of Q-analysis whereby patterns of q-connectivity are analyzed rather than connectivity (Combinatorial Connectivity in Social Systems; an application of simplicial complex structures to the study of large organizations, 1977). The approach has been applied with varied success as discussed by Jacky Legrand (How far can Q-analysis go into Social Systems Understanding?, 2002) and S. B. Seidman (Relational Models for Social Systems, 1987). Examples include: P. Doreian (Polyhedral Dynamics and Conflict Mobilization in Social Networks, 1981) and L. C. Freeman (Q-analysis and the Structure of Friendship Networks, 1980).

In a later work (Ron Atkin, Multidimensional Man: can man live in three dimensions? 1981) he develops a theory of polyhedral events and makes a convincing argument that structural events are related to clock time in a non-linear way related to their dimensions. He gives a convincing explanation why higher dimensional events take a lot longer to occur in clock time than simple events. Jeffrey Johnson (Hypernetworks for Reconstructing the Dynamics of Multilevel Systems, 2006) develops this argument that polyhedral dynamics form trajectories in a non-linear way in clock time. The formation of a simplex is a polyhedral event. Polyhedral events mark the passage of system time. Events occur at different levels on multilevel systems, and they have to be coordinated. Johnson's argument is that:

Networks are fundamental for reconstructing the dynamics of many systems, but have the drawback that they are restricted to binary relations. Hypergraphs extend relational structure to multi-vertex edges, but are essentially set-theoretic and unable to represent essential structural properties. Hypernetworks are a natural multidimensional generalisation of networks, representing n-ary relations by simplices with n vertices. The assembly of vertices to make simplices is key for moving between levels in multilevel systems, and integrating dynamics between levels. It is argued that hypernetworks are necessary, if not sufficient, for reconstructing the dynamics of multilevel complex systems.

The relevance of Q-analysis to psycho-social issues is discussed in a commentary on the Global Strategies Project (Comprehension: social organization determined by incommunicability of insights). It is also discussed in Beyond Edge-bound Comprehension and Modal Impotence: combining q-holes through a pattern language (1981).

**Polyhedral theory**

Polyhedra, linear inequalities and linear programming can be seen as three views of the same concept. Polyhedra represent a geometrical point of view, linear inequalities represent the algebraic point of view, and linear programming the optimization point of view. As noted by Geir Dahl (An Introduction to Convexity, Polyhedral Theory and Combinatorial Optimization, 1997)

Today, optimization problems arise in all sorts of areas; this is the age of optimization as a scientist stated it in a recent talk. Modern society, with advanced technology and competitive businesses typically needs to make best possible decisions which e.g. involve the best possible use of resources, maximizing some revenue, minimize production or design costs etc.... The large amount of applications, combined with the development of fast computers, has led to massive innovation in optimization. In fact, today optimization may be divided onto several elds, e.g. linear programming, nonlinear programming, discrete optimization and stochastic optimization. In this course we are concerned with discrete optimization and linear programming.... Typically these are problems of choosing some "optimal subset" among a class of subsets of a given nite ground set. Many of the problems come from the network area, where nding a shortest path between a pair of points in a network is the simplest example.

It is the identification of the closure associated with appropriate "convex polyhedra" that is closely associated with identification of an "optimal subset". Most problems studied in combinatorial optimization involve looking for certain structures in graphs. The simplest symmetrical polyhedra (octahedron, dodecahedron, etc) may therefore be understood as representing or "containing" the solutions to challenges of optimization in many fields. As Dahl further notes:

Well, polyhedral combinatorics is an area where one studies combinatorial optimization problems using theory and methods from linear programming and polyhedral theory... Polyhedral theory deals with the feasible sets of linear programming problems, which are called polyhedra. Now, polyhedral theory may be viewed as a part of convex analysis which is the branch of mathematics where one studies convex sets, i.e., sets that contain the line segments between each pair of its points.

**Polyhedral computing: optimizing responses to complexity**

The set of feasible solutions of a linear optimization problem is a convex polyhedron. Specially structured variants of these problems define polytopes with special structures. In this way the theory and algorithms of linear optimization are inherently linked to polyhedral theory and properties of convex bodies.

The generic issues posed by the above concerns were brought to a focus at the Centre de recherches mathématiques (University of Montreal) in the form of a semester on Polyhedral Computation Combinatorial Optimization (2006) presented as follows:

The last 15 years have seen significant progress in the development of general purpose algorithms and software for polyhedral
Polyhedral computation addresses the computational complexity of solving problems associated with convex polyhedra and search for efficient algorithms. One of the most fundamental problems is the vertex enumeration problem that is to list all vertices of a convex polytope given as the solution set to a system of m linear inequalities in d-variables -- as succinctly presented by Jakub Marecek (Polyhedral Approach to Multicriteria Optimization, 2006). It is in this sense that polyhedral approaches are fundamental to decision-making under uncertainty, if only in the field of economics (Andreas Eichhorn, et al. Polyhedral Risk Measures and Langrangian Relaxation in Electricity Portfolio Optimization, 2005).

More generally it is worth considering the possibility that an appropriate polyhedral configuration of criteria, opportunities and constraints constitutes a comprehensible, credible "solution" to any strategic dilemma. The argument here, notably in the light of the "epistemic" framing provided by Roth (above), is that it is precisely the polyhedral coherence (through symmetry effects) that renders a solution both understandable and memorable as a gestalt that "works". Thus what is otherwise understood through the intuitionist school of mathematics is echoed in how a viable solution is apprehended beyond that discipline. This possibility is clearly of significance to governance and to that to which the governed are asked to subscribe -- and to the manner in which a complex solution can be communicated (notably in the light of Aikin's arguments regarding the communicability of insights).

How should governance envisage optimizing the multicriteria decision-making challenges of the future -- given the computing resources on which it can call -- in a manner to render the "solutions" comprehensible and credible to all concerned?

In purely material terms, such arguments are of course consistent with the aesthetic appeal of sacred geometry as a response to architectural and design challenges. The question is how such polyhedral configuration might prove significant in psycho-social organization and knowledge structure design, notably when it takes virtual form on the web (cf Sacralization of Hyperlink Geometry, 1997; Spherical Configuration of Categories -- to reflect systemic patterns of environmental checks and balances, 1994; Spherical Configuration of Interlocking Roundtables: internet enhancement of global self-organization through patterns of dialogue, 1998).

Polyhedral methods of conjoint analysis

Conjoint analysis, also called multi-attribute compositional models or stated preference analysis, is a statistical technique that originated in mathematical psychology. Today it is used in many of the social sciences and applied sciences including marketing, product management, and operations research.

Olivier Toubia (New Approaches to Idea Generation and Consumer Input in the Product Development Process, 2001) describes a method, dependent on computer support, for an adaptive question design method that attempts to reduce respondent burden while simultaneously improving accuracy. For each respondent the question design method dynamically adapts the design of the next question using that respondent's answers to previous questions. The adaptive method interprets question de-sign as a mathematical program and estimates the solution to the program using recent developments based on the interior points of polyhedra.

Toubia begins with a conceptual description that highlights the geometry of the conjoint-analysis parameter space, permitting analyses of decision challenges involving many "dimensions" -- understood as distinct features of a product (3 to 100, say) which respondents are called upon to evaluate. The polyhedral method is designed to "shrink" the feasible set of features -- reducing its dimensionality -- determining the key features of the product design. The respondent's answers to the first q questions define a (p-q)-dimensional hyperplane which intersects the initial p-dimensional polyhedron to give a (p-q)-dimensional polyhedron, namely one of lower dimensionality. The challenge is to select questions that reduce the dimensionality of the polyhedron as fast as possible.

Conjoint analysis is one of a set of techniques of multivariate analysis in which, for example, potential customers are asked to compare pairs of products and make judgements about their similarity (or their dissimilarity in the case of ordination statistics). By contrast, whereas techniques such as factor analysis, discriminant analysis, and conjoint analysis obtain underlying dimensions from responses to product attributes identified by the researcher, multidimensional scaling obtains the underlying dimensions from respondents' judgements about the similarity of products rather than being dependent on researchers' judgments (in furnishing a list of attributes to be shown to the respondents). The underlying dimensions then emerge from respondents' judgements about pairs of products making it the most common approach to perceptual mapping.

Polyhedral design of computer memory utilization processes

The power of supercomputers is partly due to their use of a design based on a hypercube configuration of distributed memory parallel computers (see N-dimensional modified hypercube). The plurality of nodes or cells is interconnected to provide a shared memory with processors of the network and their memory providing the network routing and shared memory. Distributed memory parallel computers offer both the potential for a dramatic improvement in cost/performance over conventional supercomputers.

Of relevance here is the application to resource intensive computations in fluid dynamics required by applications, for example:

- COMET: incorporating mathematical models of a wide range of thermo-fluids and solids phenomena, powerful solvers enabling full complex-geometry capabilities, highly efficient and stable set of numerical solution algorithms, flexible pre-processing and
post-processing facilities. With no restriction on the shape of cells, grids are automatically optimized for complex geometries to achieve maximum accuracy with minimum effort. When compared with simple tetrahedral meshes, polyhedral meshes offer the same accuracy with typically half the memory requirement and a quarter of the computing time.

- STAR-CD: a simulation tool for computational continuum mechanics and computational fluid dynamics. The combination of high quality automatic polyhedral mesh generation and efficient solver technology provides improved ease-of-use, faster turnaround times and increased solution accuracy, for all classes of flow, in all application areas.

Combinatorial computational geometry: The primary goal of research in combinatorial computational geometry is to develop efficient algorithms and data structures for solving problems stated in terms of basic geometrical objects: points, line segments, polygons, polyhedra, etc.

There is tremendous pressure to develop computers of higher performance in response to certain modelling challenges, notably those related to climate and ocean currents and to challenges of astrophysics and fundamental physics. But it is curious that the applications of relevance to psycho-social challenges have been limited to relatively simplistic forms of "global modelling". There has been no implication that there might be equally demanding applications, of equivalent significance for humanity, associated with building richer and more complex psycho-social systems.

**Polyhedral databases: operational significance**

The efficiencies associated with memory process organization in computers have given rise, as an example, to databases that reflect these efficiencies. On of these is Polyhedral which is used as a key component in the infrastructure of an embedded application where data management is a considerable factor in the design. The application brings together the benefits of SQL database technology with a powerful set of high performance features designed specifically for the embedded market. The product is used in mission critical military systems (Lockheed Martin UK Selects Enea's Database Management Systems for Merlin Helicopter. Military Embedded Systems, 2007).

Polyhedral uses a memory-resident design that boosts performance by up to an order of magnitude relative to conventional disk- and flash-based RDBMSs. The active, event-driven technology makes databases more robust, simplifies applications and enhances performance. The active query mechanism allows applications to be kept up to date without the need for applications to poll the server (to detect changes). Because the client application is told precisely what has changed, the application need not refresh its queries. This all leads to a responsive system with low latency data distribution, and good scalability with graceful degradation in times of peak load.

Presumably such techniques will shortly prove to be significant to hand-held computer access to the next generation of the web -- driven by commercial innovations -- irrespective of any consideration of the potential psycho-social significance of such developments.

**Polyhedral patterns: representation of complex numerical abstractions**

As a consequence of the computer-related developments above, a polyhedral pattern library has been instigated by Roberto Bagnara at the University of Parma (Roberto Bagnara, Convex Polyhedra for the Analysis and Verification of Hardware and Software Systems: the "Parma Polyhedra Library", 2003; Roberto Bagnara et al., The PPL: A Library for Representing Numerical Abstractions: Current and Future Plans, 2004).

The Parma Polyhedra Library (PPL) is a modern and reasonably complete library providing numerical abstractions especially targeted at applications in the field of analysis and verification of complex systems. The PPL can handle all the convex polyhedra that can be defined as the intersection of a finite number of (open or closed) hyperspaces, each described by an equality or inequality (strict or non-strict) with rational coefficients. The PPL also handles restricted classes of polyhedra that offer interesting complexity/precision tradeoffs. The library also supports finite powersets of (any kind of) polyhedra and linear programming problems solved with an exact-arithmetic version of the simplex algorithm.

As highlighted by the PPL (The Parma Polyhedra Library), justifications for the creation of such a library include:

- programs need to be designed, developed and maintained over their entire lifespan (up to 20 and more years) at reasonable costs;
- programs have exploded in size over the last 25 years so that more and more with tens of millions of lines of code are in general use;
- unassisted development and maintenance teams do not stand a chance to follow such an explosion in size and complexity;
- the problem of software reliability is one of the most important problems computer science has to face;
- this justifies the growing interest in mechanical tools to help the programmer reasoning about programs;
- the large number of bugs in much of software is becoming intolerable, even in office applications, but highly publicized examples include:
  - the Mars Climate Orbiter burned in the martian atmosphere in 1999 after missing its orbit insertion because unit computations were inconsistent. The same year, Mars Polar Lander is suspected of having crashed on Mars upon landing when a software tag was not reset properly,
  - in the 1997 Mars Pathfinder (MPF) technology demonstration mission, a day's exploration time was lost when ground support teams were forced to reboot the system while downloading science data.
  - NASA's 2003 Mars Exploration Rover (MER) mission includes two rovers. At a cost of $400 million for each rover, a coding error that shuts down a rover overnight would in effect be a $4.4 million mistake, as well as a loss of valuable exploration time on the planet.

Again the question is on what range of polyhedral patterns is psycho-social organization currently based and whether much of relevance could be extended that range. Does the restriction of the range considered credible result in what Magoroh Maruyama has termed "subunderstanding" (Polyocular Vision or Subunderstanding, 2004)? In his terms is "polyocular" ensured by "polyhedral"?
Polyhedral networks: strategic significance

As noted above, network robustness (Anthony H. Dekker and Bernard D. Colbert, Network Robustness and Graph Topology, 2004) is a significant concern in the increasing focus on net-centric warfare (D. Alberts, J. Garstka and F. Stein, Network Centric Warfare: developing and leveraging information superiority, 1999; John Arquilla and David Ronfeldt (Eds.). Networks and Netwars: the future of terror, crime, and militancy. RAND Corporation, 2003).

Dekker and Colbert make very clear the strategic advantages of particular polyhedral configurations of networks. Dekker has also explored related concerns in military performance (Network Topology and Military Performance, 2005; Agility in Networked Military Systems: a simulation experiment, 2006).

The earlier paper (Towards Polyhedral Global Governance: complexifying oversimplistic strategic metaphors, 2008) raised the question of whether a polyhedral approach would help to reframe the essentially "stuck" and obsolete approaches to governance of recent decades. It is clear that such approaches are considered vital to more complex forms of decision-making, notably as faced by business and the security services. Presumably the challenges and opportunities of governance are not otherwise understood to be complex, nor to offer windows of opportunity which more complex approaches could detect -- as they are used to detect marketing opportunities for consumer products.

It is curious that no consideration has apparently been given to their application to the supposedly critical challenges of global governance. The question was raised elsewhere as to why significant segments of leadership seemingly subscribed to what amounts to a "flat-earth" approach to the strategic challenges of the future (Irresponsible Dependence on a Flat Earth Mentality -- in response to global governance challenges, 2008).

The situation does however result in a multiplicity of governance initiatives wandering the plains of such a flat Earth and competing for resources -- a metaphor of the challenge that the peoples of the Earth collectively face. It is for this reason that there is a case for experimenting with the configuration of the set of such initiatives together as a polyhedron in order to suggest more global patterns of significance from their complementarity (see Configuring Global Governance Groups: experimental visualization of possible integrative relationships, 2008).

Polyhedral relationship networks?

Whilst emphasis may be appropriately placed on the larger challenge of global governance and the new approaches seemingly required, it is useful to be aware of the many challenges faced by family and interpersonal relationships at this time. The incidence of family violence and divorce are but indicators of this. It could however be argued that the inadequacies of personal relationships are reflected in the inadequacies in governance relationships. As remarked by Gregory Bateson: We are our own metaphor.

As noted above, there is a blossoming of awareness of social network relationships -- especially beyond the family. Extended family networks, and kinship networks, remain of significance to many. Beyond the partners often desperately sought, individuals tend to be very focused on supportive networks of friends.

The question might be asked as to whether, other than the somewhat arbitrary structure of such "networks", are there other ways of understanding those relationships that might enhance a sense of well-being and identity to a greater degree. Is the coherence and quality of a set of relationships enhanced by what is effectively its configuration as a polyhedron of some form -- which may evolve or transform in response to circumstances, as with any sense of team?

Given the complexities of the relationships within which many are embedded, would such configuration enhance the kind of desirable outcome to which family therapists may aspire? Alternatively, what is it that gives significance and a sense of well-being to an ordered pattern of relationships -- beyond the hierarchical, patriarchal or matriarchal models?

Viable projects and businesses make much of the elaboration of an appropriate "business model". Is it possible that the set of polyhedra might highlight unforeseen patterns of relationship -- as "psycho-social models" -- that would enhance the meaning of those so engaged and empowered? One concrete approach is that explored by Stafford Beer (Beyond Dispute: the invention of team syntegrity, 1994) based on the icosahedron -- and subsequently subject to restrictive licensing. Inspired by Fuller, the concept of a self-organizing geodesic democracy has, for example, been developed in a series of documents by Roan Carratu (The Geodesic Direct Democratic Network; Structure, Process; Modes; Finances; General Archives; Projects; Growth; Details of Specific Procedures, 2005) and on an associated website on geodomocracy. Like syntegrity, it focuses on small group processes icosahedrally organized -- but set here in a larger context (see explanation of Geonet, 2000). Is there a much larger range of possibilities to be explored from which many might benefit -- if only in the learnings to be derived from their explorations of them? The explosion of social networking offers an ideal context for such exploration -- with an early proposal providing an interesting example (Group Questing or Twelving: proposal for a large-scale small-group development process, 1976).

Given that understandings of transparency and its desirability in configurations of relationships, there is a case for exploring this metaphorically through new understandings of glass -- especially given the widespread strategic use of optical metaphors (perspectiove, focus, vision, etc). Glass may be understood as an inorganic product of fusion, cooled to a rigid condition without crystallizing. Whereas the solid state of all known metals and metallic alloys consists of regular, periodic arrangements of the atoms, a metallic glass consists of metallic atoms arranged in a random manner with no obvious long-range correlation in the atomic positions. These new have proved to be of considerable technological importance for their unique magnetic, mechanical and corrosion-resistance properties. It might be asked whether this suggests a way of thinking about potentially valuable new configurations of psycho-social relationships.

The work of Subir Sachdev (Icosahedral Ordering in Supercooled Liquids and Metallic Glasses, 1992) focuses on the structural properties of dense and supercooled systems of atoms interacting with each other through spherically symmetric forces. He finds that...
there are significant short-range orientational correlations between the atomic arrangements applicable to any dense, supercooled liquid of spheres interacting with a pair-potential which has a repulsive hardcore and a weak long-range attraction -- readily modelled in computer. Perhaps ironically a key property of the systems considered is that they are "frustrated". Sachdev remarks:

By 'frustration' we mean that particles in the ground state cannot simultaneously sit in the minima presented to them by pairwise interactions with their neighbors. This leads to a large degeneracy in the ground state. In phase space, the system has large numbers of nearly equal free energy minima separated by substantial free energy barriers. If the system gets locked into one of these minima upon cooling from the liquid, a glassy or amorphous state can result. Our main objective shall be to determine the atomic correlations at a 'typical' local minimum of the free energy.

Sachdev's discussion of how the the "frustration" of flat three dimensional space is relieved, by appropriate minimum-energy icosahedral packing of polyhedra, points to possibilities of ordering relationships in new ways.

**Polyhedra-based sense of identity?**

Elsewhere (Emergence of Cyclical Psycho-social Identity: sustainability as "psycically" defined, 2007) in a discussion of Interlocking cycles as the key to identity it was suggested that:

Intuitively it would seem clear that it is not single cycles or unrelated cycles that provide a framework for identity in time. Perhaps the most relevant explorations of these matters are those undertaken or inspired by R Buckminster Fuller (Synergetics: explorations in the geometry of thinking, 1975-79). However his emphasis, as an architect/designer, is on "systems" that are too readily understood as static -- whatever he may have intended, and however much his work emphasizes energy systems.

With a somewhat different emphasis, his concept of a minimal "system" can be understood as minimally comprehensible "identity". His thorough exploration of polyhedral and tensegrity spatial structures, and their definition by different patterns of interlocking circles, also lends itself to interpretation in terms of temporal structures -- or at least spatio-temporal structures. The interlocking circles -- notably when well-named as "great circles" -- are then to be understood as interlocking cycles.

In this sense one or more polyhedra may prove to be a suggestive template or scaffolding onto which to project identity or with which to associate or order it. The sense of identity is then carried to a significant degree by the symmetry of the polyhedron and the manner in which the facets, edges and vertices complement each other -- whatever content is mapped onto them. Speculatively, the individual facets might then be understood as distinct "windows" on the world -- through which the world is engaged and through which identity is reinforced. Metaphorically the facets might be understood as having lens-like optical characteristics, serving a "polyocular" function in Magoroh Maruyama's terms, in bringing reality into focus. Each polyhedron then constitutes an array of such facets through which the world may be engaged (Spherical Configuration of Categories -- to reflect systemic patterns of environmental checks and balances, 1994).

Identity understood dynamically in this way might then also have its correspondence at the collective level.

**Missing link: self-reflexive closure?**

The polyhedral approaches presented above do indeed point to possibilities in relation to psycho-social organization. Missing however is any extensive exploration of how the polyhedral form can be cognitively decoded as a meaningful mnemonic. The work on epistemic networks is only suggestive of such possibilities. The work at the Cognitive Engineering Lab (University of Western Ontario) by Jim Morey and Kamran Sedig on an Archimedean Kaleidoscope as a "a cognitive tool to support thinking and reasoning about geometric solids" (Interactive Metamorphic Visuals: exploring polyhedral relationships, 2001) indicates other possibilities [explore their applet]. The focus is however on rendering polyhedral comprehensible and not on the cognitive processes of which they may constitute a formal representation.

How is it that "network" proved so intuitively appealing as a metaphor? What is it that makes forms like yantras, mandalas or rose windows so appealing (to some) as patterns or indicators of significance? What is it that elicits such appreciation within analogous 3-dimensional architectural spaces? This was the specific focus of Christopher Alexander and his team (A Pattern Language, 1977; The Timeless Way of Building, 1979) in identifying patterns that offered a subtle sense of a desirable "place to be" or a "sense of place" -- of feeling "at home", as discussed separately (Patterns as enabling emergence of a "quality without a name"). From a design perspective this is a sense of "goodness of fit" -- a concept shared with decisions or solutions meeting complex criteria in several relevant polyhedral approaches reviewed above.

The question is then the nature of the psychological engagement -- or identification -- with the configuration of criteria that constitutes closure on an acceptable solution. What in fact is the psychological appeal of a "solution" as a kind of strange attractor -- akin to "coming home" (Human Values as Strange Attractors: coevolution of classes of governance principles, 1993)? How is that cognitively embodied (cf George Lakoff and Mark Johnson, Philosophy in the Flesh: the embodied mind and its challenge to western thought, 1999; E. Thompson and F. J. Varela, Radical Embodiment: neural dynamics and consciousness, 2001)?

Curiously it may be that aesthetics, notably poetry, offers pointers to pattern embodiment -- especially in relation to governance (Poetry-making and Policy-making: arranging a marriage between Beauty and the Beast, 1993). George Hart (The Pavilion of Polyhedrality) associates distinct pieces from J. S. Bach with polyhedral images -- recalling Johannes Kepler's Harmonice Mundi (1619).

The missing link might then be understood as the cognitive processes implicitly mapped in the steps of polyhedral approaches to...
decision-making. The challenge might be framed in terms of the "cognitive decoding" of those steps -- as so clearly identified by Jakub Marecek (Polyhedral Approach to Multicriterial Optimization, 2006). It is somewhat ironic that the most comprehensible articulation of the process (found during this literature review) is that of Olivier Toubia (New Approaches to Idea Generation and Consumer Input in the Product Development Process, 2004) with respect to polyhedral methods for adaptive choice-based conjoint analysis in market research - especially the discussion of polyhedral question design and estimation.

Whilst links and nodes are widely understood as metaphors -- notably in relation to the web -- what about faces, planes, cuts, cones and point location? What of the processes of truncation and stellation -- and the implications of complementary duals?

What form do these decision-making steps intuitively take in real life, independently of the mathematical formalization with which Marecek and others are so familiar? Where is the evocative wording that would build the vital bridge between the riches of formalization and the intimacy of the experiential encounter with decision-making?

What of the closure that highlights the importance of convex polyhedra? Is it characteristic of a degree of self-reflexivity?

How might such closure on a solution relate to the intuitive understanding of a "settlement" achieved by an appropriate configuration of "stakeholders" -- exploiting the ambiguity of these terms in relation to both decision-making and architectural construction of a "safe and sheltered communal space"? Any exploration of the architectural metaphor could usefully take account of the role of "compression" and "tension" elements that enable material construction and their implications for construction of cognitive spaces, existential "places to be" and even "comfort zones" (Groupware Configurations of Challenge and Harmony - an alternative approach to "alternative organization", 1979). These are issues at the interface between tensegrity structures and polyhedra (as mentioned above). The challenge might be framed in terms of the question: what sort of network feels like home or like a community? Again, Christopher Alexander's design pattern language offers a suggestive template for any cognitive analogue (see 5-fold Pattern Language, 1984; Governance through Patterning Language: creative cognitive engagement contrasted with abdication of responsibility, 2006).

Does intuition offer access to decisions arising from polyhedra in more than 3 dimensions -- as suggested by Ron Atkin (1981)? To what degree are solutions associated with n-dimensional polyhedra meaningful? Is it polyhedra of this kind that offer coherent, comprehensible solutions to the dilemmas of sustainable development?

It is this that would give a sense to what might be experienced in a "polyhedrally empowered" network -- how it "hangs together" comprehensively as a viable, communicable solution. Also of potential significance is the implication that it is crystals, as natural forms of polyhedra, that are capable of reflecting and refracting light -- not networks. Whilst "crystalization" is considered inherently problematic in psycho-social organization, it may prove to be of unsuspected significance in a "supersaturated" knowledge society (see separate discussion Patterning Archetypal Templates of Emergent Order: implications of diamond faceting for enlightening dialogue, 2002).

Beyond the use of "Windows" as an operating system with an associated interface to the web, there is a certain irony to the parallel between the array of wall-mounted displays characteristic of:

- strategic "war rooms" enabling tactical information to be configured into strategic knowledge
- governance situation rooms (Heiner Benking, Situation Rooms: situation spaces, scales, proportions, patterns, and consequences in perspective, Research Institute for Applied Knowledge Processing, 1996; Robert W. Lamson, Planning and Using Situation Rooms, 1981)
- enterprise virtual situation rooms (W. J. Tolone, Virtual Situation Rooms: connecting people across enterprises for supply-chain agility, 2000)
- recreational facilities (pubs and cafes) on which a range of programmes are presented for clients, and notably to promote particular products and perspectives

The evolution of this arrays into wrap-around (effectively polyhedral) cognitive experiences suggests a strong link to the design challenges of "cognitive fusion" in which disparate information is brought into focus (Enactivating a Cognitive Fusion Reactor Imaginal Transformation of Energy Resourcing (ITER-8), 2006). The screens then function as cognitive lenses justifying exploration of optical metaphors.

Such explorations might then offer a new approach to strategic dilemmas, using polyhedral forms as a guide to strategic "goodness of fit" as a pattern in a design sense. This would then have important implications for governance. In building this bridge, it is worth recalling the early words of Harold Lasswell (The Transition toward more Sophisticated Procedures, 1968):

Why do we put so much emphasis on audio-visual means of portraying goal, trend, condition, projection, and alternative? Partly because so many valuable participants in decision-making have dramatizing imaginations ... They are not enamoured of numbers or of analytic abstractions. They are at their best in deliberations that encourage contextuality by a varied repertory of means, and where an immediate sense of time, space, and figure is retained (In: D B Bobrow and J L Schwartz (Ed). Computers and the Policy-making Community: applications to international relations)

Symmetry: competition vs complementarity?

The argument above points to the value of symmetry both in ensuring network robustness and comprehensibility of complexity. However, from a complexity science perspective as argued by Chris Lucas (personal communication):

Symmetry when applied to social networks seems to me to be a problematical basic concept. Whilst in ideal worlds a democracy implies a power symmetry, in real worlds such a state is unstable and rapidly becomes asymmetrical in as many ways as can be humanly found! Given human nature, any 'fair' distribution soon suffers from the 'Tragedy of the Commons' and becomes
There is a fruitful play on the relationship between "dual" and "jewel" in Development. Elsewhere it was suggested that sustainable related paper (polyhedrally empowered). Of particular interest are the dynamics of possible relationships between changing classificatory framework. Team players with specific roles engage. This argument is obviously valid in those terms. However it does not take account of dynamics of polyhedral empowerment. Such a management team may arise if particular roles are missing or over-represented. This would suggest that in this sense the nodes in a "polyhedrally empowered network" are empowered precisely because they constitute an ordered diversity of skills or perspectives -- a viable system in the sense advocated by R Buckminster Fuller (synergetics: explorations in the geometry of thinking, 1975/1979) and by Stafford Beer (Beyond dispute: the invention of team syntegrity, 1994).

Intriguing in the case of the spherical symmetry explored above is the sense that some nodes, often half, are not visible from any perspective when the polyhedron is viewed. They may be understood to be in a "shadow" zone. The sphere has to be rotated to bring them successively into view. This suggests a way of thinking about "otherness", namely "them" rather than "us" -- typically a challenge to be met, possibly through the dynamics of competition.

The interplay between complementarity and competition is of course most evident in the archetypal symmetry of the relationship between man and woman. The challenge, exemplified in that context (and by the challenges of comprehending spherical symmetry), is how to internalize the "other". Another metaphor, using the dual form of polyhedra, is the implication of the possible need to alternate into the dual form, or to allow for its expression -- a transformation well-illustrated by morphing (Carl Erikson, Morphing Three Dimensional Polyhedral Objects, 1994; Wayne Carlson, et. al. Shape Transformation for Polyhedral Objects, 1992).

With respect to dynamics, Chris Lucas again argues:

I have no problem with the intuitive appeal of symmetry or the appeal of mapping social networks onto polyhedra, one advantage is that making all links explicit will show up just how little of the network is taken into account in normal decision processes (weak links or links from weak nodes are usually ignored). Null links are links too of course, so symmetry in this sense is the superset of asymmetry.

But the dynamic issue is perhaps a killer. It seems to me that any mappings (even partial or asymmetrical ones) do seem to lock in the network to a static form. In real social networks the dynamics are ever changing, so the nodes, links, values and strengths cannot be regarded as static "variables" and if allowed to vary the "network" rapidly diverges from the model. Taking a model that no longer resembles the reality and "solving" it is a very unscientific practice (even if so very common in number-crunching circles).

This argument is obviously valid in those terms. However it does not take account of classic examples such as the resonance hybrid dynamics of the benzene molecule so fundamental to the organic world. Again it does not take account of how symmetrical arrays of team players with specific roles engage with one another in football. Nor does it take account of the archetypal dance between man and women. Of great interest in relation to the above references to Galois lattices, is the lattice focus of the work of Patrick Heelan (Logic of changing classificatory framework, 1974).

Of particular interest are the dynamics of possible relationships between polyhedral forms. As with a strategic play in football, a polyhedrally empowered network might may morph into another array, or between several arrays, as indicated in the images in the related paper (Configuring Global Governance Groups: experimental visualization of possible integrative relationships, 2008). Elsewhere it was suggested that sustainable development was perhaps to be understood as based on alternation (Policy Alternation for Development, 1984).

Polyhedral empowerment: "eliciting the sparkle from networks"?

There is a fruitful play on the relationship between "dual" and "jewel" in that the wealth of civilization -- its highest values -- may be
epitomized not so much by its precious stones (often set in symbolic "crowns", rings, or pendants) as by the dynamics of duality, so well mapped by morphing polyhedra. Hence also the association with sacred geometry as emblematic of the highest forms of integration and order.

It is also interesting to recognize that networks as such do not "sparkle"; it is the crystals, associated with the polyhedral forms into which networks may be folded, that enable the reflection and refraction of light for which they are so valued. Eliciting this capacity, in its most enhanced forms in gems and jewels, is dependent on symmetry (Steven Dutch, Symmetry, Crystals and Polyhedra, 1999). More specifically gems are most notably associated with the set of polyhedral forms that exhibit a degree of self-duality amongst themselves, namely the 5 Platonic forms: tetrahedron, cube/octahedron, icosahedron/dodecahedron. Enhancing their capacity to sparkle is achieved by appropriate cutting, faceting and polishing -- themselves offering interesting metaphors with respect to networks.

Crystals are:

- formed by linking of coordination polyhedra of which the most common are: 4-fold (tetrahedral), 6-fold (octahedral), 8-fold (cubic) and 12-fold (close-packed); more unusual forms are also possible.
- grouped into one of 7 crystal systems based on the symmetry of their atomic arrangements: cubic (isometric), tetragonal, hexagonal, rhombohedral (trigonal), orthorhombic, monoclinic and triclinic. The symmetry of the arrangement of atoms is associated with the regular geometric form of many (uncut) crystals.
- classified in 7 crystal systems in terms of the set of 14 Bravais lattices, namely categories of symmetry groups for translational symmetry in three directions, or correspondingly, a category of translation lattices. In 2 dimensions there are five Bravais lattices. (rectangular, centered rectangular, hexagonal, and oblique); in 4 dimensions, there are 52 Bravais lattices (of these, 21 are primitive and 31 are centered).

As noted above, gems offer an interesting metaphor regarding processes of relevance to governance (Patterning Archetypal Templates of Emergent Order: implications of diamond faceting for enlightening dialogue, 2002). In the above context, where polyhedral nets embody the necessary symmetry to enable them to "come alive" and "sparkle", what is to be said of the portions of the network that are appropriately configured to ensure that this process occurs?

- are the factions forming a polyhedrally empowered network to be understood as an appropriately configured array of facets, each important to this process?
- what is to be understood about how the polyhedral form is "cut" to expose the facets most capable of enhancing "sparkle"?
- given the etymological root, is "cut" then to be understood as related to "decision", and to decision-making capacity?
- what is to be understood by the necessary asymmetry required in a gem?

Given the way in which reflection and refraction work in gems, the existence of gem-like forms in which light is only reflected from the outer surface offers interesting metaphors regarding those in which light may only be reflected between the inner surfaces, not allowing it to pass out -- what might be termed "dark crystals". The issue might be related to the distinction between bonding ("within crystal") and bridging ("between crystals") (cf Dynamically Gated Conceptual Communities: emergent patterns of isolation within knowledge society, 2004; Y. Connie Yuan Geri Gay, Homophily of Network Ties and Bonding and Bridging Social Capital in Computer-Mediated Distributed Teams, 2006). This is partially played out, notably in academic circles, through what have been termed "mutual citation packs".

Presumably such issues of reflection and refraction could be understood as being intimately related to the sense of image -- especially self-image -- and the importance attached to it by a person or a group.

Given the features of a network, people (or groups) may be variously identified with (and hold responsibility for):

- a node or vertex (perhaps representing a faction or a perspective)
- an edge or link (perhaps representing a challenging or sympathetic relationship to another)
- a face (perhaps representing a coalition, minimally of three parties, but increasingly unstable beyond 6 unless appropriately set in a more complex polyhedral configuration)
- a great circle linking the nodes (perhaps to be understood as a theme or discussion thread) defining a plane of symmetry of a spherically symmetrical polyhedron
- an axis of symmetry perpendicular to any plane of symmetry (perhaps to be associated with cyclic forms of identity, as mentioned above)
- the polyhedral pattern as a whole, notably arising from the interlocking of the various thematic great circles, each of different orientation corresponding to different axes of symmetry (as discussed in Spherical Configuration of Interlocking Roundtables: internet enhancement of global self-organization through patterns of dialogue, 1998)

Recognition of curvature as fundamental to a polyhedral psycho-social universe?

Networks and Sponges: In dealing with the frustration of undertaking strategic initiatives, the omnipresence of networks is frequently acknowledged through exasperated phrases such as "everything is connected to everything". It is therefore appropriate to note that sponge structures are the most abundant forms in nature, on all possible scales of material existence. But, as noted by Michael Burt (Periodic Sponge Polyhedra: expanding the domain, 2007), the amount of morphological insight into the phenomenon is meager. It may therefore be appropriate to see unforeseen aspects of polyhedrally empowered networks in terms of insights into the ordering of sponges -- as a generalization of the order conventionally associated with polyhedra based on planar surfaces. This is achieved by Burt through the introduction of the curvature of faces and edges -- an intuitively reasonable expansion of perspective.

"Curved" psycho-social relationships? It might be considered curious that psycho-social relationships, bonds and links are commonly...
conceived as "linear" -- whether or not they are considered multi-valent or as polyhedral nets. They may indeed have a quality of "curvature" as is often implicit in any understanding of their dynamics.

Burt notes:

Lately, after confronting the prevalent definitions and allowing for polyhedral maps with curved edge-lines and face surfaces, the amount of uniform sponge polyhedra exploded, to reveal a multitude of new polyhedral sponge configurations, spherical, toroidal and hyperbolic, and their governing hierarchical order. The new sponge imagery might play a significant role in the morphological research of natural bio-forms and physical nano-structures, promote images and ideas of innovative space structures and influence the way we perceive our increasingly dense urban habitat... With some extrapolation of the perceiving mind it is right to claim that the sponge phenomenon, with its porosity and permeability characteristics, is central to the physical morphological nature of the human habitat, on the urban and the building scale, and represents its defining imagery.

He does not mention their possible relevance to psycho-social order, but presumably would not exclude it, as implied elsewhere (Michael Burt, Periodic Sponge Surfaces and Uniform Sponge Polyhedra in Nature and in the Realm of the Theoretical Imaginable) in stating:

Fig. 1. Periodic Sponge Polyhedra: expanding the domain

Spherically symmetrical radiolaria have long offered an appeal to the imagination as organic patterns with which psycho-social organization might be in some way isomorphous. The quality of sponginess, as noted by Burt, has been recognized in many languages as the description of a physical phenomenon characterized by porosity and visual permeability whose equivalences are recognized in "porous" psycho-social organization -- and, pejoratively, in the dynamics between some people.

Given the different degrees of approximation of the regular polyhedra to the curvature of the circumsphere on which their vertices are located -- and the intuitive significance of a sphere as emblematic of desirable coherence and wholeness -- there is a case for envisaging the concave spaces within sponge-like forms as encompassing forms.

Symmetry, topology and combinatorics: Burt describes his approach to the spongiform domain as follows:

The employed exploratory process involved a certain fusion of symmetry and topological reasoning and combinatorics. It was clear from the outset that the domain of the periodic sponge polyhedra is dominated by certain topologically categorized classes: the Spherical; Toroidal; Hyperbolical and, what could be aptly categorized as Primitive (Cyclic; Stripe; Columnar and helicoidal) P.S.P.s. All these Periodic Surfaces and Polyhedra correspond to various symmetry groups which account for the uniformity of the vertex configurations. (Periodic Sponge Polyhedra: expanding the domain, 2007)

Primary Parameters of the Polyhedral Universe. The parameters found to be most suitable by Burt to describe the polyhedral phenomenology in 3-D space are the following:

1. Genus-g: which determines the Euler characteristic, K=2(1-g) of the various 2-d manifolds on which the topologically different polyhedral maps are drawn. Each 2-d manifold, and all the potentially possible polyhedral maps on it will be differentiated as belonging to a particular g=n domain....

2. \( \sum a \ av. \): the average sum of polygonal angles in a vertex, and more generally- the total (average) curvature value pertaining to a single vertex region of a polyhedral envelope. In case of uniform polyhedra, the \( \sum a \) is the same for all vertex regions of the polyhedron and is equal to the total curvature of its 2-d manifold subdivided by the number of its vertices....

3. Valav.: the average valency of the polyhedral mosaic (the average number of edges emanating from a vertex), inspired by the Periodic Table of the Elements, representing chemical bonding, is not even hinted by Descartes's and Euler's theorems. It was revealed after a prolonged exploratory process that any meaningful collection of polyhedra, if arranged according to the growing valency values, show a consistent evolution of (most of) their secondary parameters (V, E, F, average polygon-Pav., stability, and so on). Generally speaking, Val. represents density of the polyhedral lattice and has a critical bearing on stability of the lattice, when realized as a physical bar and joint space frame....

Periodic Table of the Polyhedral Universe (Michael Burt, 1996): This remarkable synthesis has been subsequently described by Burt (Periodic Sponge Surfaces and Uniform Sponge Polyhedra in Nature and in the Realm of the Theoretical Imaginable) as to some extent a visual animation of Euler's formula of \( V-E+F = 2(1-g) \), producing a medium through which all its apparent and hidden meanings are revealed.

Burt's synthesis is constructed on the basis of thorough selection of the above primary parameters. These:

seem to capture the essence of the polyhedral- topological nature, and when used as coordinates of a Cartesian 3-D space, provide for an environment in which every conceivable 3-D polyhedron has a unique point representation. It goes without saying, that due to space combinatorics and permutations, the same parameters of Val, \( \sum a \) and g might be shared by more than one polyhedron, thus giving rise to the notion of "isotopes"; i.e. polyhedra sharing same location within the "Table's" space.
Of critical importance therefore, when dealing with the polyhedral universe and especially with its "sponge domain", is the determination of these primary parameters on the basis of which, as coordinates of a Cartesian space, the Periodic Table of the Polyhedral Universe is constructed. This then provides a domain in which every conceivable polyhedron has a unique point representation and discloses patterns of polyhedra sharing various geometric-topological characteristics, related to these primary parameters. It constitutes a powerful research tool of the polyhedral universe. Burt further notes:

The "Table" is bridging over the g-domain borders of spherical \((g=0)\) polyhedra, the toroidal \((g=1)\) and the hyperbolic \((g\geq2)\) sponge like polyhedra and brings the whole polyhedral universe into a unified perspective. All conceivable polyhedral phenomena of topological association and similarity relationships, between various polyhedral groups, phenotypes and families, describable in terms of the primary parameters, support location patterns, on the basis of which a comprehensive classification and hierarchical structure of the entire polyhedral universe can be constructed. These location patterns, some of which are universal (namely embracing all the polyhedral universe), and some of which are limited, local, unique or esoteric, describe by proxy, polyhedral properties of varying significance and scope, and their relative distribution within the coordinated and highly structured space.

What is visually (and intellectually) striking is the realization that while all the accumulated rich imagery of finite-spherical and the toroidal polyhedral mosaics are limited to \(g = 0\) (spherical) and the \(g = 1\) (toroidal) domains and \(\sum \alpha = -2\pi \chi 2\pi\); the sponge polyhedral imagery stretches over all \(g\)-domains \((g\geq2)\), with \(g\), \(\sum \alpha\) and Val. values reaching to infinity.

Burt (2007) concludes:

When perceived through the coordinated space of the Periodic Table of the Polyhedral Universe, it becomes clear that with the addition of the sponge polyhedra domain, the emerging polyhedral universe is exploding to cosmic dimensions, as compared with the Ptolemaic world picture of the polyhedral domain of just one decade ago. When all the horizon of sponge structures is taken in, it dawns on us that the number of... [Periodic Surfaces and Polyhedra]... with all their phenotypes, clans and families, many of which include infinite number of members, each, is overwhelming, much in excess of all the familiar polyhedra in the \(g = 0\) (spherical) and the \(g = 1\) (toroidal) domains. So, it's not just in the natural - physical, but also in the theoretical and possibly imaginable world of geometry that the sponge configurations constitute the overwhelmingly greater majority of shapes and forms.

Isomorphism relevant to psycho-social organization?: It is curious that Burt's work on the fundamentals of order has emerged from the design-focused context of a university faculty of architecture and town planning -- just as has the past and recent work on a pattern language of Christopher Alexander and his team (A Pattern Language, 1977; The Timeless Way of Building, 1979; The Nature of Order, 2003-2004). In the absence of analogous insights into the fundamentals of order specific to psycho-social organization in a knowledge society -- rather than for its "shelters" and "settlements" -- these approaches would seem to constitute a strong justification for the exploration of their implications for new understanding of more appropriate patterns of psycho-social and cognitive organization.

The focus on sponges, given their essentially organic and biological nature (despite their interest for architects), highlights in a remarkable manner the inappropriateness of an essentially inorganic and abstract assumption regarding networks. Discussion of networks of all kinds typically focuses on their abstraction as linear relationships between nodes -- themselves treated as abstractions. This is necessarily true of polyhedral nets and of the polyhedra they may form in three dimensions (despite their significance for psycho-social organization as argued above). Absent from any such discussion is the existential participatory experience (in the moment) within any such network. Such networks are at best treated like telecommunication networks of wires between devices -- of which the organic equivalent is the nervous system. This is far from the experience of such a network -- to which Atkin's work so notably points.

Embodying an experiential dimension: There is a seemingly unexplored approach to representation of networks, and their configuration into polyhedra, which relates it to Burt's work on sponges. It may be recognized that:

- any link (in a network or as the edge of a polyhedron) can have diameter -- namely forming a tube (rather than a mathematical abstraction or an electrical wire) -- creating a volume of cognitive space consistent with the coherence of the experience of movement between nodes. Curiously "links", as abstractions, typically de-emphasize the psycho-social implications of their "length" in any representation. This is notably the case with respect to hyperlinks where the potential connectivity and instantaneous emphasized through telecommunication that obscures any psycho-social sense of "distance", "alienation", "delay" or "learning time".

- any node (in a network or as the vertex of a polyhedron) can have diameter -- namely forming a sphere (rather than a mathematical abstraction) -- a volume of cognitive space is created consistent with the coherence of the distinct entities linked in the network.

In this way the tubes of the network link experiential spaces in a manner somewhat similar to the manner in which any sponge is organized. There is a certain irony to any focus on the humble "sponge" (in all its variety, as identified by Burt) potentially offering a more organic insight into the experiential significance of networks, whether polyhedrally configured or not.

Further support to imaginative reflection on the complexities of psycho-social organization is offered by the possibility of allowing the diameter of either the link (edge) or the node (vertex) to increase considerably, and even disproportionately, whilst retaining the geometry of the polyhedron. Possibilities include:

- Vertices increase in radius such that their spheres:
  - "kiss touch" with neighbouring spheres: In this case the contact is tangential with the edge (of minimum diameter)
linking the centres -- and of zero length (indicative of zero "distance" between them)

- **contact through a circular interface with neighbouring spheres:** In this case the diameter of that circle might be understood as the diameter of the link between them -- again of zero length
- **envelope the sphere of the opposite vertex:** An interesting condition in which any vertex (A) of a polyhedron is represented with a radius that ensures that the vertex (B) most distant from it in the polyhedron is on the surface of a sphere centered on (A). When all vertices are represented in this way -- and with all such spheres necessarily overlapping -- a sense of the cognitive space associated with each distinct node is offered. This is notably helpful when part of the challenge lies in the differing extent to which any such entity subsumes the experience of others in the network -- possibly challenging the very nature of their relationship.

- **Edges differ in diameter from the vertices such that:**
  - **when equal in diameter** (or slightly exceeding) that of the vertices, there is an implication that virtual forms ("avatars" in cyberspace parlance) of a vertex can roll through a tubular structure of tunnels, perhaps usefully modelling psychological projection
  - **when less in diameter** than that of the vertices, there is an implication that virtual forms of a vertex can move along the structure of links like beads on a three dimensional "abacus" of relationships.

Curiously the abacus has been discussed with respect to the polyhedral arrangement of molecules by A. G. Samuehson (*A Molecular Abacus for Electrons*, 2001) reporting on the work of Eluvathingal, D. Jemmis, et al. (*A Unifying Electron-Counting Rule for Macropolyhedral Boranes, Metallaboranes and Metallocenes*, 2001) on a new electron counting scheme for three-dimensional cages linked together. As Samuelson notes: "Apart from the fact that it accommodates old rules, and explains difficult structures, it is satisfying to note that Jemmis has rationalized the new rules using fragment molecular orbital theory". Jemmis notably specializes in fullerene molecules (known as "buckyballs" in their spherical form).


Perceived from a given vertex, the intersection with another vertex can be understood as a form of Venn diagram (or one of its variants), whether in 2 or 3 dimensions -- suggestive of "cognitive sponges". Further possibilities include:

- **Changing diameters:** Here the diameters change over time, possibly with a degree of periodicity, to constitute a dynamic geometry -- but conserving the polyhedral invariance defined by the central elements (ignoring the possibility of morphing to other polyhedral forms). This is suggestive of shifting emphasis, or the perception of shifting emphasis. However it is also usefully reminiscent of the dynamics of life sponges.

  - **Concentric elements:** Here each feature, whether (tubular) edges or (spherical) vertices, is understood as having additional concentric forms. These might usefully model different "levels" of the polyhedral representation of psycho-social relationship. Levels might be typically understood to include: physical, emotional, conceptual, intuitive or spiritual.

**Decision-making as changing orientation:** The introduction of curvature by Burt highlights interesting perspectives within a psycho-social context in which "horizon effects" are necessarily a feature of ignorance and denial -- as effectively recognized by Atkin. Any "linear" progression over a convex curved surface eventually results in the starting place becoming "out of sight" as the effect of curvature comes into play -- especially problematic if curvature is denied.

To the extent that spherically symmetrical polyhedra approximate to the curvature of a sphere as they increase in complexity, there is a case for recognizing that decision-making at a nodal point (a vertex) is associated with a change of orientation -- from that of one link (edge) to any others meeting at that node. A vertex is then a point at which the various decision options are presented -- all of which will prove (after linear pursuit of their orientation) to be suboptimal with respect to the integrity associated with the circumsphere on which the vertex is located. [N.B. This postpones any discussion of the potential significance of stellation -- precluding the association of some vertices with the circumsphere. It also postpones discussion of the contrasting integrity that might be associated with the insphere -- the largest sphere that is contained wholly within the polyhedron, being dual to the dual polyhedron's circumsphere.]

There will be preferences (vigorously) articulated for one orientation rather than for another. Typically the focus is on selecting the "right" decision for further "forward" progress. This may be understood as either:

- deliberately avoiding any deviation from the preferred great circle pathway in favour of an "alternative". This may then be understood as a learning pathway with each successive node as the culmination of a learning phase, perhaps associated with a critical challenge
- deliberately switching to an alternative experiential pathway. Here the node may be understood as the temporal condition in which the inadequacies of the current orientation come to a focus. This may be framed as the search for a complementary orientation or simply as a richer distraction. Use of a TV remote control to select amongst a set of channels for a better programme illustrates the experience.

However the polyhedral framework highlights the interrelationships amongst the set of disparate orientations (and the interlocking great
circles) that effectively defines the system as a whole -- whether to be understood as a psycho-social ecosystem, an "ego-system", or even as a dialogical "echo-system". It suggests a degree of fruitless inappropriateness in seeking to persuade all of the inherent superiority of any particular great circle pathway (Spherical Configuration of Interlocking Roundtables: internet enhancement of global self-organization through patterns of dialogue, 1998). This is especially the case where, as a learning pathway, each is best understood in temporal terms, as a temporal learning/action cycle (notably as explored by Arthur Young, The Geometry of Meaning, 1976, summarized in Characteristics of phases in 12-phase learning-action cycle, 1998). The challenge of achieving significant consensus is then epitomized by that of ensuring fruitful multi-generational, multi-cultural (community) gatherings -- given the experiential and orientational differences.

Following the discussion above with respect to a polyhedrally-based sense of identity, and elsewhere (Emergence of Cyclic Psychological Social Identity: sustainability as "psyically" defined, 2007), any challenge of consensus might then be best understood as one of reconciling an array of "voices", each developing a distinct theme from one vertex to another -- in song. This is the musical challenge of eliciting harmony from polyphony and multi-part singing in large choirs. It highlights the cultural and cognitive significance traditionally associated with song and rhythm, as notably explored from a philosophical perspective by Antonio de Nicolas (Meditations through the Rg Veda, 1978) and Ernest G McLain (The Myth of Invariance: the origins of the gods, mathematics and music from the Rg Veda to Plato, 1976; The Pythagorean Plato: prelude to the song itself, 1978).

The underlying polyhedral structure, whether temporal or otherwise, may then be understood as a "meta-pattern" in terms of the central thesis of Gregory Bateson (Mind and Nature: a necessary unity, 1979):

> The pattern which connects is a meta-pattern. It is a pattern of patterns. It is that meta-pattern which defines the vast generalization that, indeed, it is patterns which connect.

It is from this perspective that he warns in a much-cited phrase: Break the pattern which connects the items of learning and you necessarily destroy all quality.

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