Psycho-social Significance of the Mandelbrot Set

a sustainable boundary between chaos and order

Annex 2 to Sustainability through the Dynamics of Strategic Dilemmas -- in the light of the coherence and visual form of the Mandelbrot set (2005)

Introduction

The concern in this Annex to Sustainability through the Dynamics of Strategic Dilemmas is to indicate features associated with the Mandelbrot set (hereafter the M-set) in order to point to their significance in configuring complex experience -- rather than in describing natural phenomena, as is normally the case. The assumption is that the features offer templates for innovative thinking in response to highly divisive strategic and value dilemmas. An assumption is also made that the mind is uniquely capable of undertaking operations that explore the features of complex spaces such as those with which the M-set is associated. Note that contextual arguments and references are provided in the main paper.

Such an assumption is reinforced by the documented abilities of inventor Nikola Tesla (1856-1943). He is known for having envisioned, built and tested complex electromechanical devices in his mind (without blueprints) -- with a degree of success, even to sensing when they were out of balance. For this reason he did not even build prototype models. As he stated in his autobiography: "The moment one constructs a device to carry into practice a crude idea, he finds himself unavoidably engrossed with the details of the apparatus." Furthermore: "As he goes on improving and reconstructing, his force of concentration diminishes and he loses sight of the great underlying principle." Tesla's inventions invariably assembled together without redesign and worked perfectly. (Nikola Tesla: Humanitarian Genius)

There is therefore every possibility that the essential properties of the M-set, or particular features, have been discovered through non-mathematical disciplines -- notably those concerned with the development of meditative concentration techniques. This possibility is highlighted below. The possibility has even be documented in a delightful trap for the unwary (Ray Girvan, The Mandelbrot Monk, 1999).

The concern of this argument might therefore be expressed in terms of responding to the question as to how someone -- with such a degree of insight -- might then choose to communicate those insights more widely. And how would they do this in the absence of the requisite mathematical techniques or corresponding computer graphics abilities?

Would they seek to hang the insights onto: the structure of the human body (the chakra system); the human heart (the "sacred heart" of the Aztecs and the Christians); the cross (of Christianity); a mandala; a container or vessel (as in alchemy); an arrangement of trigrams (the Ba Gua mirror); temple design (sacred architecture); etc? In other words is there some kind of a structure that effectively acts as an "index" to such insights -- whilst in itself pointing to a level of subtle experiential coherence that interrelates them without subsuming any of them? What form might such a "Rosetta stone" of the subtlest experiential insights take?

The question is therefore the degree to which the following features of the M-set can be meaningfully and usefully internalized as a template for valuable new forms of thinking.

Potential implications in terms of religious symbolism
- **Traditional posture of meditation:** The similarity of the form of the M-set with the representations of the meditating Buddha has been frequently noted (see above). It is somewhat amusing that the conventional representation of the M-set, with the "head" on the left, is that of the so-called "reclining Buddha" -- and only some fractal browsers allow the M-set representation to be reoriented. Such a shift in orientation does however point to difficulties of comprehension -- given the conventional attribution of the x and y axes in mathematics in general, and in representations of the complex plane in particular. It is of course the case that the graphical representation of the M-set, even re-oriented, may vary according to preferences for colouring. This is a useful reminder with regard to the distracting nature of representation. It is however worth noting the striking resemblance of some representations of the M-set with some representations of buddhic deities (notably Tibetan) with a flame-like aura.

- **Configurations of numerous Buddhas:** One of the remarkable features of the M-set is its fractal nature. As a consequence of a degree of self-similarity, within the M-set there are to be found infinitely numerous replicas of the buddha-like shape of its representation as a totality. However Tibetan Buddhist mandalas may show a central Buddha-figure, surrounded by smaller secondary Buddhas -- each of which may in turn be surrounded by even smaller Buddhas. In this respect it is worth recalling that in Buddhism the Buddhas are traditionally recognized to be "as numerous as the sands of the Ganges". They would appear to be ordered in a manner reminiscent of the organization within the M-set, as illustrated by the following extract from a Vajraguru prayer on *The Resolve to Practice Excellence*:

  > The force of my resolve to practice excellence brings all victors clearly to mind; emanations of my body, numerous as the atoms of all universes, bow down in perfect obeisance before them. All Buddhas, numerous as the atoms of all Buddha realms, stand in a single atom, surrounded by their children. Similarly, this host of victors stands in every single atom throughout the realm of totality. To them I direct my devotion and faith. [more]

This suggests interesting explorations of the number of Buddhas in relation to the number of "iterations" required to resolve their isomorphs as details within the M-set -- especially in the light of understandings of the necessarily lengthy cycles of "reincarnations".

- **"Fantastic" imagery:** The jewel-like quality of some of the "fantastic" detail of the M-set is also echoed in Buddhist texts such as *The Lotus Sutra: The Mystic Powers of the Tathagata*:

  > The living beings in their midst, the heavenly beings, dragons, yakshas, Gandharvas, asuras, garudhas, kimnuras, mahoragas, human and nonhuman beings, thanks to the Buddha's supernatural powers, all saw in this sāhu world immeasurable, boundless hundreds, thousands, ten thousands, millions of Buddhas seated on lion seats under the numerous jeweled trees... Moreover, they saw immeasurable, boundless hundreds, thousands, ten thousands, millions of bodhisattvas and mahasattvas....

As noted by Stanislaw Lem (*The Cyberiad, 1975*), and cited in a *study* of the M-set:

> Everyone knows that dragons don't exist. But while this simplistic formulation may satisfy the layman, it does not suffice for the scientific mind....The brilliant Cerebron, attacking the problem analytically, discovered three distinct kinds of dragon: the mythical, the chimerical, and the purely hypothetical. They were all, one might say, nonexistent, but each nonexisted in an entirely different way....Suppose, for example, one organizes a hunt for such a dragon, surrounds it, closes in, beating the brush. The circle of sportsmen, their weapons cocked and ready, finds only a burned patch of earth and an unmistakable smell: the dragon, seeing itself cornered, has slipped from real to configurational space.

- **"Heavens" and "Hells":** Both Hinduism and Buddhism postulate the existence of a multiplicity of hells and heavens -- or lower and higher worlds -- variously characterized by continents, oceans and other natural features. These may well be reminiscent of details encountered in zooming into graphic representations of the M-set, possibly depending again on the colouring technique used. Although perceived as "ridiculous" to western rational thought, there is the intriguing possibility that the identification and naming of these regions (or the myriad deities above) within some belief systems is as legitimate as a naming procedure for dynamic features of chaotic space as the naming of celestial objects by astronomers.

- **Engendered features:** Some religions, notably Hinduism and Buddhism, stress the degree to which the features of the perceived world, including "heavens" and "hells", engendered by the mind -- as a product of the potential of "mindlessness" or "no mind". The graphic detail of the M-set -- widely noted as variously reminiscent of rivers, leaves and other natural phenomena -- is also effectively engendered through the M-set.

- **I Ching trigram arrangement (Pa Kua):** As noted above with respect to Taoism, the 8 basic trigrams of the *I Ching* or *The Book of Changes* are conventionally displayed in a circular manner (known as the *Ba Gua Mirror*). As discussed below, the trigrams respectively correspond to dynamics that are collectively fundamental to sustaining coherence as indicated by the understanding of Tao.

With respect to the M-set, these distinct dynamics may be associated with the features of different period, termed primary "bulbs", that are attached to the cardioid region of the M-set representation. How this is done depends on insights into the two classical arrangements of importance to Feng Shui, the *Ho tu* or *Lo Shu* [more | more | more]. A relationship between the Tao and...
the Mandelbrot set seems to have been noted by Katya Walter (A New (and very Old) Model for Nonlinear Computation, 1995), a Spanish group [more]. It has been explored to some degree by Arnold Keyserling (The I Ching and the Five Stages of Creative Time, 1999):

Self organization can be understood as the capacity to create information based on the Mandelbrot vector... considered as a continuous creative process. The key is spontaneous improvisation in the moment -- acting in the Tao -- returning to Zero. This whole field has been thoroughly explored for millennia by Chinese thinkers and so we will use their terminology. The underlying principle of self organization, the factor creating a larger identity, is called chi in Chinese philosophy (ki in Japanese). (Strangely enough, this Chinese word is the same as the Greek word for the Pythagorean symbol.) Chi appears in two aspects, wu chi, emptiness, symbolized by the empty circle, and tai chi, plenitude, symbolized by the ancient Chinese fractal...

Extending the trigram system to the 64 hexagrams of the I Ching, and based on Mandelbrot's fractal theory, analogies have been explored between the traditional Chinese energy system on the one hand, and the 64 units of the genetic code or any other energy system on the other (see Cornelius Celsus Foundation, Energy-Based Deciphering of the Genetic Code). A more detailed comparison in terms of both Mandelbrot and Julia sets has been made by Tony Smith (3x3 Octonion Matrix Physics Models).

Fractal patterns fundamental to occult representations of reality: In addition to Buddhism (as noted above), other beliefs systems may explicitly indicate the fractal nature of reality.
- **Kabbalah**: In this Jewish occult system everything can be classified in terms of, and reduced to, the ten Sefirot -- fundamental archetypal principles or essences, qualities of the manifest and knowable Godhead. Each of the ten Sefirot that together make up creation is itself composed of ten Sefirot, each of which is in turn composed of ten. Each of the Sefirot is thus divided fractal-wise into sub-sefirot, and those into further sub-sub-sefirot, so the same applies with the doctrine of worlds. [more]
- **Astrology**: In a careful articulation of the relevance of nonlinear dynamics to understandings of astrology, Michelle Jacobs (Bringing It Down to Earth: a fractal approach, 1995) argues:

> If Chaos theory is so important to modeling the behavior of complex systems -- that is, the behavior of the natural world, then perhaps it has a similar role in the complex workings of astrology, for astrology also reveals a subtle relationship between simplicity and complexity; it too initiates life. Astrology and Chaos both provide means of plotting the unfolding of processes in time and they each allow one to get a holistic overall view of nature.

But although Jacobs refers to the M-set, her argument is developed in relation to strange attractors:

> In Chaos theory, when we look at the large picture, apparently random events can be shown to happen within patterns, with the Strange Attractor representing the overall predictable state. And in astrology we can easily see the birthchart as a Strange Attractor: It outlines the pattern of the life and personality, but within that pattern there are infinite variations on the theme. Truly, each sign or planetary pattern contains a full potential of self-similar or fractal expressions, but these expressions never fall outside the attractor. An astrological planet contains all of its possible presentations within the overall predictable form of the planetary symbol.

- **Temple design**: Sacred architecture is often premised on the insight that it will reflect the subtlest of insights as a temple of the spirit [more].

- **Cross (Christian)**: As noted earlier, the cross may be associated with the axes that enable the graphic representation of the M-set in a complex plane. The junction of the vertical and horizontal elements then lies at the origin. It is interesting however how the choice of orientation is not arbitrary. The seated-Buddha orientation is associated with an inverted cross -- a symbol associated with satanic rituals. Such dramatic contrasts call for further reflection (see below) on the content associated with the "real" and "imaginary" dimensions of the complex plane and how "right", "left", "above" and "below" are distinguished -- and especially how values are projected onto them. Of great interest to such an exploration is the work on arithmetical epistemology of Xavier Sallantin (1975). His work on the nature of "evil" from that perspective also merits careful attention (Le problème du mal à la lumière de la cyberscience, 2001). Also of interest is the relation of the symbolism of the inverted cross to that of the upward-pointing sword. It is such challenges that point both to the role of the cross in relation to the resolution of fundamental paradoxes and to the tensions held by the cross and embodied in the dynamics encoded by the M-set.

- **Sacred heart (Christian, Aztec)**: The cardioid, as noted earlier, can be very explicitly associated with the "sacred heart". Like the cross, it is also inverted in relation to the seated-Buddha orientation. The cardioid as a complex mathematical object is of the greatest interest in holding a very wide variety of relationships as noted in an earlier paper (Cardioid Attractor Fundamental to Sustainability, 2005). As noted earlier, any exploration of the dynamics of the geometric construction of the cardioid suggests a rich pattern of associations supportive of insights associated with any meditation on the "sacred heart". Of particular interest, for example, is the cardioid evolute (see Figure 2) -- which, by contrast, is conventionally oriented within the "seated" orientation of...
In much religious symbolism there is an effort to "hang" subtler understanding onto the features of the body -- as with the classic image by Leonardo da Vinci of the outstretched body across a circle, as a measure of all things (known as Vitruvian Man). There are numerous references to the subtle isomorphism of God and human:

- **Body as the temple of the spirit**: "Do you not know that you are a temple of the Spirit, whom you have from God, and that you are not your own?" (1 Corinthians 6:19) and the need for cleansing it [more]. Emphasis may be placed on each "saved" person functioning as such a temple. For Christians the body of Jesus was a temple of the Spirit (John 2:19-21). In the Essene Gospels, Jesus indicates: "the body is the temple of the spirit, and the spirit is the temple of God. Purify, therefore, the temple, that the Lord of the temple may dwell therein and occupy a place that is worthy of him". Many religions, and martial arts, teach that the body is the vessel, the "temple" of the spirit.

- **Image of God in man**: This is a theme widely explored [more | more | more] and especially by Robert C. Newman (Some Perspectives on the Image of God in Man from Biblical Theology, 1984). For Christians, for example: The imago Dei, the image of God in man, is first mentioned in connection with man's creation on the sixth and final day of creation -- "So God created man in His own image; in the image of God He created him; male and female He created them." (Genesis 1:27-28) [more]

**Potential mytho-poetic implications**

Insights distinguishing levels of abstraction, of the same subtle order as the M-set, may have been deliberately (or inadvertently) anchored in cultural artefacts such as myths, legends, leys and the like. Whether they are termed "archetypes" or not, their fundamental nature and importance is nourished by the societies by which they are valued for the order they offer.

Examples of the possible relationship between such cultural artefacts and the form of the M-set include:

- **Two dragons chasing a pearl**: The decorative art of China, including temples and traditional folk dances, focuses extensively on the theme of two dragons. They are usually depicted facing one another in the air in eager pursuit of a spinning pearl floating like an iridescent bubble between them. This theme was a mark of books issued under imperial auspices. For Taoists, the complex associations of that pearl include wisdom, yang energy, truth and life -- even the everlasting life of those who perceive the truth and attain enlightenment. The pearl can also be thought of as a symbol for universal Qi energy -- the progenitor of all energy and creation. The two dragons are associated with an understanding of eternity. The Chinese have a belief that "descendants of the dragon". Given their "protuberances and jewelled scales", the two dragons can readily be associated with the two sides of the M-set representation -- in pursuit of the circular 2-period ball.

- **Two dragons of Arthurian mythology**: These supposedly represent the two aspects of the British nation: the red one Celtic and the white Saxon.  

- **Ba Gua (Pa Kua) and associated dragons**: The 8 basic trigrams of the I Ching, in their traditional circular disposition (discussed below), have also been associated with the nine classic dragons of China [more]. Each of these symbols can be understood as guarded by dragons, each with distinct personalities and powers. As a set they protect the insights represented by the cardioid region of the M-set.

- **"Here be dragons"**: It is perhaps trivial to recall that ancient maps had a tendency to mark the outskirts of maps of the known world with phrases like "here be dragons". It is indeed the case that what can be known, or "remembered", in any permanent sense might be said to be represented by the boundary of the M-set.

- **"Sword in the stone"**: This basic Arthurian myth regarding the sign of true royalty might well be associated with the understanding of how to distinguish, or grasp, the sword-like axes effectively embedded within the chaotic stone-like structure of the M-set representation.

- **"Withdrawing into the stones"**: This Irish Celtic myth recalls the story of an elder race (the Tuatha Dé Danaan) that through their wisdom were eventually able to "withdraw into the stones". There is now extensive modern myth-making in relation to "intraterrestrial" at another "vibratory level". Do some implications of mediational insight suggest that people can withdraw into the "fabric of reality" as embodied in the dynamics encoded by the M-set?

In each case, these myths might also be understood as an ability to function at the level of abstraction mapped by the domain of the M-set, again to be understood as a form of stone.
Potential experiential implications in terms of concentration and meditation

It is possible that the M-set usefully points to the challenges of concentration and meditation. Indeed it is possible that it was those with such skills that were instrumental in formulating myths to anchor their insights as an aid to wider understanding.

- "Oceanic awareness": This feeling, frequently reported by investigators of alternative states of consciousness, might well be associated with the cardioid portion of the M-set -- or quasi-similar sub-variants of it.
- Mindlessness: This condition (also termed "no mind"), to which meditators of some spiritual disciplines aspire, might be associated with the level of abstraction of the cardioid portion of the M-set. As expressed from a Kriya Yoga perspective, for example, in commenting on the Bhagavad Gita:

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Understanding of Gita happens in our bone marrow and blood-cells through repeated chanting and contemplation. Gita invites us to be available to the holistic consciousness of choiceless awareness, that is, to the 'ignorance' which knows -- not to the 'knowledge' which is ignorant! Gita is the unique world view that tolerates and requires holding together multiple positions simultaneously so that religion holds whole mankind and does not degenerate in bigotry and battle. The war in Gita is symbolic. It is the war between wickedness and wisdom, between mindlessness and 'no-mind' with mind as the bridge. Duryodhan is mindlessness, Arjuna is mind and Krishna is 'no-mind' i.e. pure intelligence (chaitanya). Gita liberates us from our prior self-preoccupied identities. Gita sets us free from disintegration to integration (yoga), from reaction to action (Kriya Yoga), from longing to living (swadhyay), from paradoxes to pure consciousness (Iswara Pranidhun), from Prakriti (inherent traits and tendencies) to Purusha (transcendental truth of enlightened existence). Gita is the wisdom of sacrifice of the fruits of action, the distinction of Gunas, the emergence of equanimity, and the importance of non-doership. [more]
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- Alchemy: Traditional representations of meditating practitioners of Taoist "internal alchemy" (nei-tan) bear a striking resemblance to the representation of the M-set. As discussed in an earlier paper (Cardioid Attractor Fundamental to Sustainability, 2005):

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Curiously it is Gregory Bateson in a section on Form, Substance and Difference [more] of his book Steps to an Ecology of Mind (1972) that relates the depth psychology work of C G Jung to the thermodynamics of Sadi Carnot. But it is in another book, translated by Jung's colleague Richard Wilhelm (1929), that Jung comments on a fundamental cycle identified in a Chinese text Tai I Chin Hua Tsung Chih (The Secret of the Golden Flower) -- more recently translated by Thomas Cleary (1991) -- which showed Jung how to initiate this alchemy through the breathing cycle [more | more | more]. This focus has also been compared to the Nestorian Gospel of St Thomas [more].

The much-cited Chinese work discusses the "circulation of the light" of awareness through various conditions during meditation [diagram] reminiscent (if only in the metaphors used to describe them) of stages of the Carnot heat cycle discussed above. The Taoist practitioners of "internal alchemy" (nei-tan) refer to a continual circulation of the ch'i (vitality principle) up the primary yang (positive), back or Governor Channel (tu mai or du mo) and down the primary yin (negative), front or Functional Channel (jen mai or ren mo) of the body. (see Lu K'uan Yü, Taoist Yoga, Alchemy and Immortality, 1970) [more].
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It is possible that the representation of the M-set is to be usefully related to the traditional concept of an alchemical "vessel" in which all "base matter" can be dissolved -- the container for the "universal solvent". Alchemy postulates the existence of such a universal solvent as being capable of transforming base metals into gold and bestowing eternal youth and therefore immortality on human beings. The universal solvent -- counterpart to the "philosopher's stone" -- is not ordinary water, but "philosophical" water, the water of life, aqua permanens, aqua mercurialis. For Robert Grinnell (Alchemy in Modern Woman, 1973) it is basic to the transformative alchemical process of solutio which facilitates the fluid, mobile basis of consciousness:

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For aqua permanens is a mode of the arcane substance; its symbol is water or sea-water, an all-pervading essence of anima mundi, the innermost and secret numinous in man and the universe, that part of God which formed the quintessence and real substance of Physis, at once the highest supercelestial waters of wisdom and the spirit of life pervading inorganic matter.
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For Iona Miller (Chaos as the Universal Solvent, 1993):

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In reducing all to pure water, the prima materia and the ultima materia become synonymous. That primal consciousness state, that creative and chaotic consciousness is the beginning of the operation of "water", and its ultimate realization. It becomes easy to see why the operation of water is the "root of alchemy." Through consciousness journeys which liquify our rigid notions of self and world, we re-create the adventures of the hero or heroine. The theme is the loss and recovery of identity.
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- Chakra system: In Hinduism, its spiritual systems of yoga, in some related eastern cultures, as well as in some segments of the New Age movement, a chakra (from the Sanskrit word for "wheel, circle") is considered to be an energy node in the human body [more]. The seven main chakras are described as being aligned in an ascending column from the base of the spine to the top of the head (see fractal representations). Each is associated with a certain color, multiple specific functions, an aspect of consciousness, a classical element, and other distinguishing characteristics [more]. Given the hypothesized organizing function of the M-set, it is interesting to consider where these chakras are each located in relation to the geometry of its graphical representation (in the "seated-Buddha" orientation):
It is therefore interesting to note the Buddhist comparison between such understandings and those which the M-set “embodies.”

1. **Muladhara** (Root chakra): Understood to be at the base of the human spine. On the M-set representation it is most closely associated with the junction of the inward-turned portion of the cardioid.

2. **Svadhisthana** (Sacral / Hara chakra): Understood to be below the human navel. On the M-set representation it is most closely associated with the axial origin from which the cardioid is generated. Zazen as the study of the self, is a particular kind of meditation, unique to Zen Buddhism, that is at the “very heart” of the practice. Fundamental to that practice is the centering of attention in the *hara* as the physical and spiritual center of the body. In Hinduism, this is recognized as governing the function of the gonads and the reproductive system. It is associated with the emotional body, willingness to feel emotions and accept change. As the centre of gravity of the body, this awareness is also fundamental to some Eastern martial arts.

3. **Manipura** (Solar Plexus chakra): Understood to be at the level of the human solar plexus. On the M-set representation it is most closely associated with the intersection of the vertical axis with the horizontal line between the symmetrically positioned 3-period primary bulbs attached directly to the cardioid.

4. **Anahata** (Heart/Lung chakra): Understood to be at the level of the human heart. On the M-set representation it is most closely associated with the intersection of the vertical axis with the horizontal line between the symmetrically positioned 5-period primary bulbs attached directly to the cardioid.

5. **Visuddha** (Throat chakra): Understood to be at human throat. On the M-set representation it is most closely associated with the junction of the cardioid with the circular period-2 primary bulb above the cardioid.

6. **Ajna** (Third Eye chakra): Understood to be between the human eyes. On the M-set representation it is most closely associated with the centre of the circular period-2 primary bulb above the cardioid.

7. **Sahasrara** (Crown chakra): Understood to be at the crown of the human head. On the M-set representation it is most closely associated with the crown-like features about the circular period-2 primary bulb above the cardioid.

There is a case for investigating the extent to which the position of these chakras emerges from more complex mathematical features about the cardioid, its associated curves, and especially from its generation. These might be understood as significant associative pathways for those who embody this system.

- **Types of concentration/meditation**: In the terms, for example, of the *Yoga Sutras of Patanjali* (1.17-1.18):
  All objects are in one of four stages: Virtually all types, styles, methods, or objects of meditation are included in one of these four stages, levels, or categories (1.17):
  - *Savitarka/Gross*: relates to concentration on any gross object while still accompanied with other activities of the mind, including meditation on sensory awareness, visualized objects, the gross level of breath, attitudes, syllables of mantra, or streams of conscious thought.
  - *Savichara/Subtle*: relates to subtle objects, after the gross have been left behind; the subtleties of matter, energy, senses, and the mind are, themselves, the objects of meditation, inquiry, and non-attachment.
  - *Samanada/Bliss*: emphasizes the still subtler state of bliss in meditation. In this state, the concentration is free from the gross and subtle impressions that were at the previous levels.
  - *Samadhi/I-ness*: focuses on I-ness, which is even subtler, as it relates to the I that is behind, or witness to all of the other experiences.

Objectless concentration: The four stages (above) all have an object to which attention is directed *(samprajnata)*. Beyond these four is objectless concentration (1.18), where all four categories of objects have been released from attention *(asamprajnata)*.

- **Emptiness of mind**: As with many spiritual disciplines (for which there are extensive web references [more | more]), Buddhism in its various forms (notably Mahayana Buddhism) places much emphasis on the emptiness of mind and the ways in which that understanding is obscured. Offering an intriguing association to the M-set cardioid, in the *Heart Sutra* the Buddha reportedly states, for example:

> ...form is emptiness and the very emptiness is form; emptiness does not differ from form, form does not differ from emptiness; whatever is form, that is emptiness, whatever is emptiness, that is form, the same is true of feelings, perceptions, impulses and consciousness.

In another example, the Tibetan Kalu Rinpoche reportedly stated:

> You live in illusion and the appearance of things. There is a reality, and you are that reality, but you don't know it. If you should ever wake up to that reality you would realize that you are nothing, and being nothing, you are everything. That is all. [more]

This could be understood in terms of the the human tendency to focus on, and identify with, the experience of the sensible dynamics which the M-set “embodies” — but from a higher order of abstraction with which the individual may alternatively choose to identify.

It is therefore interesting to note the Buddhist comparison between such understandings of emptiness and the origin of numbers from emptiness. For Buddhists, emptiness (*sunyata*) implies a sense of potential rather than nihilism. In *Mathematics, mind, ontology and the origins of number* the recognition by Geshe Kelsang Gyatso (1995) is noted that there are three levels of dependent relationship:

- Gross dependent relationship - causality - the dependence of phenomena on their causes.
- Subtle dependent relationship - structure - the dependence of phenomena on their perceived parts (including aspects,
• Very subtle dependent relationship - the dependence of phenomena on imputation by mind.

For Buddhists, these ideas are noted [more] as being remarkably similar to the theory of the origins of mathematics, as proposed by the mathematician John von Neumann (1923) in the light of the theory of sets. He was the originator of the architecture used in most non-parallel-processing computers. Von Neumann suggested that all numbers could be bootstrapped out of the empty set by the operations of the mind. A set is a collection of things. An empty set is a collection of nothing at all. An empty set can be thought of as nothing with the potential to become something (that is to be become a set with at least one member). Again, the three orders of abstraction might be fruitfully related to the structure of the M-set.

From a perspective of hermetic philosophy (About Spiritual Emptiness or the Void):

Emptiness designates a state of mind, an inner level of consciousness based upon the renunciation of what one believes to be real, beyond all comprehension or lack of comprehension. Emptiness is, therefore, a higher level of the mind, an attunement with "nothing," i.e., with Pure Being that has no reflection at all. This is the revelation that the abstract or higher mind of a spiritual man receives. He is called "spiritual" because his mind is open and in harmony with the whole of creation, or the Unknown God. Hence, such a man is empty of a personal "interpretation" and is in touch with the world of spirit. As he manifests the world of spirit within him, a new one opens up before him, giving him a new understanding of life.

Such references point to many possibilities of using the M-set as a means of holding the distinctions between various fundamental states of consciousness associated with "spiritual" insight. For example, given the various distinct modes of long term behaviour characteristic of the dynamical systems mapped by the M-set:

• To what extent is the notion of iterations "escaping to infinity" to be usefully associated with the typical forms of distracted everyday thinking -- and in the most extreme form of attention deficit disorder? In fact are the "number of iterations" before thinking does escape to be understood as a measure of the degree of concentration?
• To what extent is asymptotic attraction to a single value (an attracting fixed point) to be associated with "one-pointed" meditative states?
• To what extent are bounded systems characterized by periodic or cyclic patterns of a certain frequency, to be considered as associated with forms of meditation (sustained by repetitive chants) or, otherwise, by thoughts going obsessively and habitually "round and round"?
• To what extent are chaotically bound systems (neither attracted to a single value nor to a periodic cycle) to be associated with forms of concentration that are neither attracted to a particular focus nor to a periodic pattern of habitual reflection?

The extensive discussions regarding Buddhist insights into emptiness offer many pointers to such a comparison [more].

Potential implications for self-awareness, relationships and psychotherapy

• Self-referential awareness: A discussion within the Jung Circle points to the implications of fractal generation of the M-set for understandings of the self-referential nature of the psyche:

We can usefully regard this equation as the symbolic representation of the dynamics of an archetype (perhaps the Self, since it is capable of infinite magnification and reduction with no loss of detail?) Significantly, it is an iterated equation. What does this mean? Well, simply put, its form is a feedback loop (shades of the alchemical Uroboros, cyclic distillation, or the self-cycling energy of the unconscious) in which the result of each calculation (parallel with one's acquired wisdom?) is fed back into the equation as the initial value of z. [more]

• Identity: The nature of the M-set offers a means of distinguishing a higher order of identity and invariance from the various dynamics with which it may readily be confused. It clarifies that identity may indeed be associated with (a) attractors that are a fixed (and possibly obsessive) focus, (b) periodic attractors, possibly associated with habitual patterns of behaviour, (c) patterns of behaviour with a higher (chaotic) degree of order, and (d) dissipative and unconvergent patterns of behaviour. However the M-set suggests a form of psychic invariance that, whilst dependent on these dynamics, is indeed of a higher and subtler order.

How the individual becomes aware of this subtle order is presumably what many spiritual disciplines struggle to communicate and impart -- perhaps stressing the significance of particular features or dynamics fundamental to emergence of awareness of the M-set. This subtler sense of identity is presumably intimately related to a personal form of "sustainability" -- through which identity is sustained. It suggests that the "boundary" of the self is best, and most subtly, defined by the chaotic boundary of the M-set, however that is to be understood. In terms of spiritual insight, it is also worth reflecting on the possibility that the array of features of the M-set may perform functions analogous to those of an aerial in entering into resonance with yet subtler insights.

The relation between the J-sets and the M-set is succinctly indicated (see main paper) by the statement that the J-sets lie in the dynamic plane, whereas the M-set lies in the parameter plane -- in a 4-dimensional space. Adrien Douady, in a widely quoted comment, notes that "You first plow in the dynamical plane and then harvest in the parameter plane". In terms of identity it might be argued that action is necessarily in the "dynamic plane", but that identity emerges in the "parameter plane". The "mystery" of identity lies in the transformation of awareness between these two planes -- from the dynamic multiplicity of the one to the complex singularity of the other. Any incapacity in this transformation may be closely related to a lobotomy separating left-brain
With respect to identity, Dick Oliver (Fractal Vision, 1992) notes:

> When you begin to think fractal, the world is a very different place indeed. Each part of that world is defined not by its division from others, but by its resonance with a greater whole. Separation becomes a ridiculous idea when one's identity is gained by a unique transformation, rather than a greedy withdrawal. Just as every part of the body synchronizes with and permeates all others, my individuality is strengthened rather than threatened through giving and cooperation.

The questions with respect to individual identity can be extended to those of collective identity, whether at the family, community, tribal, ethnic, national or global level. What does "resonance" mean with regard to "citizenship", for example?

Any temptation to disparage the metahoric implications of such an approach to identity should take account of the teasing warning of Kenneth Boulding (Ecodynamics: a new theory of social evolution, 1978):

> Our consciousness of the unity of self in the middle of a vast complexity of images or material structures is at least a suitable metaphor for the unity of group, organization, department, discipline or science. If personification is a metaphor, let us not despise metaphors -- we might be one ourselves.

### Personal relationships:

Given the directly felt significance of attraction and love, there is a case for exploring the merit of the M-set mapping of attractors (of different types) as a way of ordering understandings of relationships between people. Clay Tucker-Ladd (The Nature of Attraction and Love, 1996) provides a useful overview.

J. C. Sprott (Mathematical Models of Love and Happiness, 2001) describes a two-dimensional linear continuous-time dynamical model of the love/hate relationship between two individuals and extends the dynamics of this simple model through some nonlinear dynamics, notably modelling love triangles with chaotic solutions.

The question is whether forms of love can be suitably distinguished in terms of types of attractors. For example, in the distinctions made by Eric Fromm (The Art of Loving, 1956):

- **Motherly love**: As an unconditional, all-protective, blissful love that cannot be controlled by the receiver, is it appropriately associated with the fixed point attractors of the cardioid region? Its absence is associated with despair, perhaps of those regions beyond the M-set boundary that are "escaping to infinity". It is currently considered as threatened by narcissism and possessiveness.

- **Fatherly love**: As a conditional love, earned through "good behaviour", is it usefully associated with the primary period-2 bulb?

- **Brotherly love**: Are the periodic dynamics of different patterns of relationships to be usefully associated with the bulbs of varifous periods? Fromm argues that brotherly love, "which underlies all others", is currently threatened by the reduction of human beings to commodities.

- **Erotic love**: Currently debased by its separation from brotherly love and the absence of tenderness.

- **Love of God**: Might be understood in terms of the total dynamic to which the M-set points. This could be consistent currently regression "to an idolatic concept of God".

- **Self-love**: Understood to be that form of love without which we cannot love others -- currently considered to be threatened by selfishness. As a form of self-reference, this this might usefully be related to the iterative process through which initial values are fed back.

There is indeed an irony to the possibility that the infinite range of dynamics of "affairs of the heart" may be, in a specially significant way, interrelated by the cardioid of the M-set representation. The evolution of "love of god" from its matriarchal form in nature religions, through a patriarchal form in monotheistic religions, points to the challenge of comprehension of the significance of the M-set in this context -- as implied by the via negativa of western mystics, and the nondualistic approaches of eastern religions.

It is of some interest that the concept of "family values" may indeed be related to the dynamics circumscribed by the M-set. The chaotic boundary zone may well correspond to those relationship dynamics of which it is said that one has to work at "making a relationship work". To the extent that the challenges of ensuring sustainable global governance are isomorphic with those of sustaining family values, there is clearly a danger in deriving simplistic concepts of global governance from simplistic understandings of the dynamics associated with family values -- as exemplified by the chaotic boundary of the M-set.

### Depth psychology:

From the perspective of depth psychology and the collective unconscious, the iterative dynamic through which the M-set is generated can perhaps be usefully associated with the concept of the Eternal Return and archetypal dynamics. As argued by John R Van Eenwyk (Archetypes and Strange Attractors: the chaotic world of symbols, 1997):

> If 'the backbone of fractals' is 'feedback and the iterator,' this may be at the heart of what Mircea Eliade called 'the myth of the eternal return.' Could iteration and the eternal return be referring to the same thing?

The eternal return begins in tensions of opposites (present and future, actual and potential, sacred and profane), manifests itself in fractal imagery (transcending categories and demonstrating self-similarity across scale through recapitulations of the original act of creation), is sensitive to initial conditions (demonstrated by the wide variety of myths and rituals of different cultures), and iterates (the eternal return). The oscillatory dynamics (tensions of opposites) that generate myths
and rituals enliven them as well, by bringing up new possibilities. Thus, creation occurs over and over again...So, the eternal return is an iterative dynamic: it allows the present to be fed back into the original equation. While all archetypal processes generate feedback dynamics, the eternal return is the epitome of all such aspects of archetypal processes. It is the archetype of archetypal dynamics, so to speak.

The M-set may be thought of as pointing to the form and nature of the outcome of the individuation process -- distinguishing precursor conditions that it might mistakenly be assumed to be, or on whose dynamics it depends. Of particular interest is the bilateral symmetry in the "seated orientation" ("real" axis corresponding to the "spinal chord"). This results from the understanding of "positive" and "negative" in relation to the "imaginary" axis -- to the right and left. This raises the possibility of associating the two halves as complementary in psychological terms, namely the challenge of integrating the shadow, as articulated by C G Jung [more] -- and the "peaceful" and "wrathful" deities of Tibetan Buddhism. The M-set is a way of expressing and holding the tension between those two halves. Of course, given the existence of "shadow cabinets", through which opposition parties articulate their views in parliaments, it points to a more general challenge of integration, including that of the "shadow of humanity".

**Four quadrant perspective of Ken Wilber:** Considerable attention has been focused on the achievements of Ken Wilber (*A Theory of Everything: An Integral Vision for Business, Politics, Science, and Spirituality*, 2000) in ordering the range of insights into understanding individual integration and personal development in a social context. He notably groups a comprehensive list of twelve "schools" of consciousness (ranging from cognitive science to psychosomatic medicine and bioenergy) as subsets of four quadrants of existence (*An Integral Theory Of Consciousness*). Central to this perspective is a model based on two axes, giving rise to those four quadrants:

- **Axes:**
  1. Individual -- Collective:
     - Individual:
     - Collective
  2. Interior view (subjective) -- Exterior view (objective)
     - Subjective: On the Left are positioned the "interior", unobservable things -- what individuals/groups think (and feel).
     - Objective: On the Right are positioned the "exterior", observable phenomena -- what an individual/society does (associated with nature and science).

- **Quadrants:** The interaction of these two axes gives four personality types, to which codes are commonly attributed:
  - Upper Left (UL): Intentional (individual subjective), notably associated with sincerity, integrity and trustworthiness (and the arts, beauty and the self).
  - Upper Right (UR): Behavioural / Neurological (individual objective), notably associated with correspondence, representation and propositional communication.
  - Lower Right (LR): Socio-economic (collective interobjective), notably associated with systems theory web, structural-functionalism and social systems mesh.(and morals)
  - Lower Left (LL): Cultural (collective intersubjective), notably associated with cultural fit, mutual understanding and a sense of rightness (and the good).

The quadrant model is enriched by concentric circles centred on the origin and indicative of successive stages of evolution or development. Thus in each quadrant there are ten or more developmental levels, such as from atoms to brains and from prehension to vision-logic. Complementing the attention that the quadrant model has attracted, a variety of reservations and criticisms have been expressed [more | more | more | more]

The question here is whether the understanding of Wilber can be meaningfully associated with the organization of the M-set, as graphically represented. Once again, the assumption of this exploration is that there should at least be a geometrical transform between the mappings they respectively constitute -- especially since the polarities identified by Wilber are fundamental to the dynamic of society.

In the case of the axes of the M-set, the distinction made (in mapping its emergence in the complex plane) is between "real" and "imaginary" -- with a concept of "positive" and "negative" in each case. It could be argued that "interior" can be understood as "imaginary", and "exterior" as "real". The problem with this is that Wilber places them on the same axis, rather than on orthogonal axes. With respect to "individual" and "collective", these might variously be understood as associated with "positive" (for those stressing individualism) or "negative" (for those stressing collectivism and community) as illustrated in the following table.

<table>
<thead>
<tr>
<th></th>
<th>&quot;Negative&quot;</th>
<th>&quot;Positive&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Individualism perceived as selfish (as typical of Asian cultures, socialism, etc)</td>
<td>Individualism perceived as the focus and justification of social development (as typical of western cultures)</td>
</tr>
<tr>
<td></td>
<td>Collectivism as perceived as a constraint on personal freedom and development (as typical of western cultures)</td>
<td>Community as perceived as the key to well-being (as typical of many non-western cultures)</td>
</tr>
</tbody>
</table>
Again it would appear that the significance of "individual" and "collective" is confused in relation to contrasting understandings -- in this case of "positive" and "negative". The Wilber and M-set mappings can possibly be reconciled by recognizing that Wilber's axes effectively run through the quadrants created by the M-set axes. In other words, rotating Wilber's axes by 45 degree offers a means of exploring their significance in relation to the M-set.

But this possible "fix" in fact points to a more fundamental issue that is of significance to further exploration of both mappings -- as with the "positive" and "negative" connotations of the orientation of the Christian cross in relation to the axes in the "seated-orientation" of the M-set representation (see above), there is a dynamic to their valuation that calls for further attention. This dynamic is associated with the a priori issues explored by mathematical epistemologist Xavier Sallantin (Xavier Sallantin, L'épistemologie de l'arithmetique, 1976) and by the direction in which an I Ching hexagram is "read" when presented with other hexagrams in any circular "arrangement" (eg out=top or in=top). The point to be explored is whether there are one or more cognitive steps between recognizing the dimensions of the complex plane in terms real (positive and negative) and imaginary (positive and negative) and the cognitive significance subsequently attached to any fundamental dimensioning of complexity in terms of individual-collective and exterior-interior.

One commentator, Andrew P Smith (Wilber's Eight-Fold Way How Many Sides Does a Holon Have?, 2004), points to some of the difficulties noted in Wilber's work:

In his most recent work, the four aspects have undergone another round of breeding. In the new generation of holons, every one of the original four aspects has an inside and an outside.... One might have thought that the terms interior and exterior would adequately convey these notions, but it turns out, according to Wilber, that interiors have both insides and outsides, as do exteriors....

Several writers have noticed that Wilber's four-quadrant model conflates two different meanings of the individual vs. social dynamic. One meaning used by Ken is the distinction between individual and social holons. The other meaning is the individual (or agentic) vs. social (or communal) aspect of any holon, individual or social.

From the perspective of his own model, Peter Collins (An Integral Mathematical Stage Model of Perspectives, 2003), notes in a contribution to the Ken Wilber Forum regarding the M-set (Fractals 2 - Recursion and Complexity):

This mistranslation of the true relationship between whole and parts (and parts and whole), I would consider the biggest problem in conventional science. Putting it more formally it represents gross mistranslation of complementary opposites (in vertical terms). Even Ken Wilber consistently mistranslates this relationship. For Ken development is synonymous with transcendence (i.e. the holarchical view of evolution where "lower" level parts are transcended and included in "higher" level wholes. However development is equally synonymous with immanence (i.e. the partarchical view where "higher" level wholes are made immanent and included in "lower" level parts). However this complementary view of development is largely missing from Ken's writings.

In a unique and careful exploration of the psychological significance of the M-set, Collins focuses his discussion on the challenge of shifting from the (conventional) quantitative appreciation of it to a holistic one. He usefully emphasizes that:

Psychological reality is indeed based on constant recursive procedures. One of the most important manifestations of this is the manner by which perceptions and concepts continually interact in experience. Every phenomenon has both perceptual and conceptual aspects. Indeed this applies intimately to such mathematical entities as numbers....

Vision-logic for example exemplifies this at the rational linear level... Here a dynamic iteration procedure continually takes place. We start with concrete phenomena. These are then transformed in conceptual terms leading to new starting phenomena in experience. And the process continues in this manner leading to an ever-changing interpretation of reality. So a psychological feedback mechanism is inherent in the very manner we experience phenomena....

Thus we now see that the psychological feedback process involves a continual movement as between (horizontal) perceptions and (vertical) concepts. The starting values are one-dimensional. In other words the perceptions are understood in linear rational terms. However the transformed values are two-dimensional. In other words the decisive qualitative shift from concrete (particular) to formal (holistic) understanding implicitly involves intuition (which combines polar opposites in experience and is two-dimensional).

Collins concludes with the vital point that: "all the important features of the Mandelbrot Set have psychological qualitative equivalents".

Potential implications: fractal quasi-similarity of patterns

The precise fractal ordering of the M-set, and the quasi-similarity of the patterns interrelated by it, raise interesting questions about the level of abstraction of the patterns perceived in any of the above cases.

This is most evident in the experiential nature of the following two cases:
• **Spiritual insight:** Given the quasi-similarity, it becomes evident how difficult it may be to distinguish between a spiritual experience with a "Buddha" -- that is effectively a sub-sub-sub variant of the whole -- and one that corresponds to an experience of the whole. The challenge lies, in the graphical metaphor, in the ability to "zoom out" beyond the limited experience to be able to view, encompass, or identify the whole. Without that, the experiencer may be "trapped" within the frame to which "browsing" has led -- perhaps righteously to be contrasted with even more partial experiences, if "zooming in" is possible. Clearly communication with others, who have encountered a quasi-similar experience elsewhere in the "spiritual fractal", may be somewhat meaningful -- but fraught with potential misunderstanding. Dogmatic statements regarding the identical nature of such experiential patterns may then be righteously made.

• **Personal relationships:** Again a pattern associated with "being in love" may in fact reflect a sub-sub-sub portion of the pattern as a whole. As such it may nevertheless be compared somewhat meaningfully with the experience of others (as in romantic encounters and fiction). But it may again be difficult to distinguish between levels of experience -- each being able to make inappropriate assumptions about the nature of the experience of the other.

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